



INDEX TRACKING IN PORTFOLIO OPTIMIZATION WITH TRACKING ERROR VARIANCE MODEL

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Abstract

Index tracking is a form of portfolio management in stock market investment. Index tracking aims to track the performance of the stock market index without purchasing all the stocks that make up the market index in order to achieve rate of return similar to the market return. This objective can be achieved by determining an optimal portfolio which minimizes the tracking error of the optimal portfolio to the benchmark market index. Tracking error is a risk measure of how closely a portfolio follows the benchmark index. The objective of this study is to determine the optimal portfolio composition by using tracking error variance (TEV) model in index tracking. The results of this study show that the optimal portfolio of TEV model is able to track the Malaysia market index which is FTSE Bursa Malaysia Kuala Lumpur Composite Index effectively with only holding 40% of the component stocks in the market index.

Keywords: Index Tracking, Tracking Error Variance Model, Mean Return, Portfolio Composition

1. Introduction

Index tracking is a popular form of passive fund management in stock market investment. Passive management is a buy-and-hold strategy that used by the investors to achieve rate of return similar to the market return [1]. Index tracking describes the process of tracking the performance of a stock market index without purchasing all the stocks in the market index [2]. The simplest way to track an index is full replication, where all of the stocks that make up the index are purchased in the same proportions as in the market index. However, this strategy is not practical since it incurs high capital. Roll [3] introduced tracking error in index tracking to reproduce the performance of the market index without purchasing all the stocks in the market index. Tracking error is a risk measure of how closely a portfolio follows the benchmark index [2]. Various optimization models for index tracking problem have been developed and studied by researchers in different stock markets [4-13]. The objective of this paper is to determine the optimal portfolio composition by using tracking error variance (TEV) model to track Malaysia stock market index. The rest of the paper is structured as follows. The next section discusses about the data and methodology. Section 3 discusses about the empirical result of this study. Section 4 concludes the paper.

2. Material and Methods

2.1 Data

FTSE Bursa Malaysia Kuala Lumpur Composite Index (FBMKLCI) is the leading indicator of the performance of the Malaysia stock market and economy which consists of 30 stocks listed on the Malaysian Main Market. In this study, the data consists of weekly return of stocks in FBMKLCI Index from January 2010 until December 2012.

2.2 Tracking Error Variance Model

Roll [3] proposed tracking error variance (TEV) model in index tracking problem. Tracking error is a risk measure of how closely a portfolio follows the index [2]. The tracking error measures the deviation of the portfolio's return compared to the market

return using variance as risk measure [14]. Index tracking is perfect if the tracking error is zero [2]. The optimal level of tracking error is less than 3.00% [15]. The objective of TEV model is to minimize the tracking error subjects to five constraints as shown below.

$$\text{Minimize } TE = \sqrt{\frac{1}{T} \sum_{i=1}^T (R_{Pt} - R_{It})^2} \quad (1)$$

Subject to

$$Z_i \in \{0,1\} \quad (2)$$

$$\sum_{i=1}^n Z_i = K \quad (3)$$

$$L_i Z_i \leq x_i \leq U_i Z_i \quad (4)$$

$$0 < L_i < U_i < 1 \quad (5)$$

$$\sum_{i=1}^n x_i = 1 \quad (6)$$

where TE is the tracking error, T is the number of periods, R_{Pt} is the mean return of the portfolio at time t and R_{It} is the mean return of the market index at time t . x_i is the weight of each stock invested, K is the number of stocks selected to track the market index. L_i and U_i are the lower and upper bounds of the investment proportion respectively on stock i .

Equation (1) is the objective function of the model which minimizes the tracking error. For constraint (2), Z_i ($i = 1, 2, \dots, n$) is introduced to indicate the stock selection problem with $Z_i = 1$ indicates the i th stock is included in the optimal portfolio or otherwise for $Z_i = 0$. Constraint (3) ensures that the number of stocks in the optimal portfolio is K . Constraint (4) shows that if stock i is not selected in the optimal portfolio (i.e., $Z_i = 0$), then $x_i = 0$, and if stock i is selected in the optimal portfolio (i.e., $Z_i = 1$), then $x_i \neq 0$. Constraint (5) indicates that the value of x_i is limited in the interval $[L_i, U_i]$. Constraint (6) ensures that the total weight of stocks invested is one.

2.3 Portfolio Performance

The performance of the portfolio is measured with tracking error in index tracking problem. Tracking error is the standard deviation of the difference between the returns of the portfolio and the returns of the benchmark stock market index. The tracking error is formulated as follow [16].

$$TE = \sqrt{\frac{1}{T} \sum_{t=1}^T (R_{pt} - R_{it})^2} \quad (7)$$

where TE is the tracking error, T is the number of periods, R_{pt} is the return of the portfolio at time t and R_{it} is the return of the benchmark stock market index at time t . Higher portfolio performance is indicated by lower tracking error.

The mean return of the portfolio over time T horizon is formulated as follow [17].

$$R = \sum_{i=1}^N R_i w_i \quad (8)$$

where w_i is the weight of each stock invested and R_i is the mean return of stock i . The portfolio generates an excess return if the return of the portfolio is higher than the return of the benchmark stock market index which is formulated as follow [13]:

$$\alpha = r_p - r_i \quad (9)$$

α is the excess mean return of the portfolio over the mean return achieved by the benchmark stock market index, r_p is the mean return of the portfolio and r_i is mean return of the benchmark stock market index.

3. Results and Discussions

Table 1 presents the results of the TEV model in selecting the stocks to construct the optimal portfolio.

Table 1: Stock Selection with TEV Model

Stock	Weights (%)
AMMB Holdings	9.19
Axiata Group Bhd	11.42
British American Tabaco	0.00
CIMB Group Holding	0.00
Digi.Com	5.75
Genting Bhd.	8.31
Genting Malaysia	3.67
Hong Leong Bank Bhd	0.00
Hong Leong Financial Group	0.00
IOI Corporation	8.52
Kuala Lumpur Kepong	0.00
Malayan Banking	11.15
Maxis Bhd	0.00
Petronas Dagangan Bhd	0.00
Petronas Gas Bhd	5.42
PPB Bank	13.81
Public Bank Bhd	0.00
RHB Capital Bhd	0.00
Sime Darby	12.46
Telekom Malaysia Bhd	0.00
Tenaga Nasional	5.79
UMW Holdings	0.00
YTL Corporation	4.52
YTL Power International	0.00

As shown in Table 1, those stocks with positive values indicate that they are selected by the TEV model in constructing the optimal portfolio to track FBMKLCI Index. The optimal portfolio consists of 12 stocks to track FBMKLCI Index which comprises 30 stocks. The components of the optimal portfolio are AMMB Holdings, Axiata Group Bhd, Digi.Com, Genting Bhd, Genting Malaysia, IOI Corporation, Malayan Banking, Petronas

Gas Bhd, PPB Bank, Sime Darby, Tenaga Nasional and YTL Corporation. Figure 1 displays the optimal portfolio composition of TEV model.

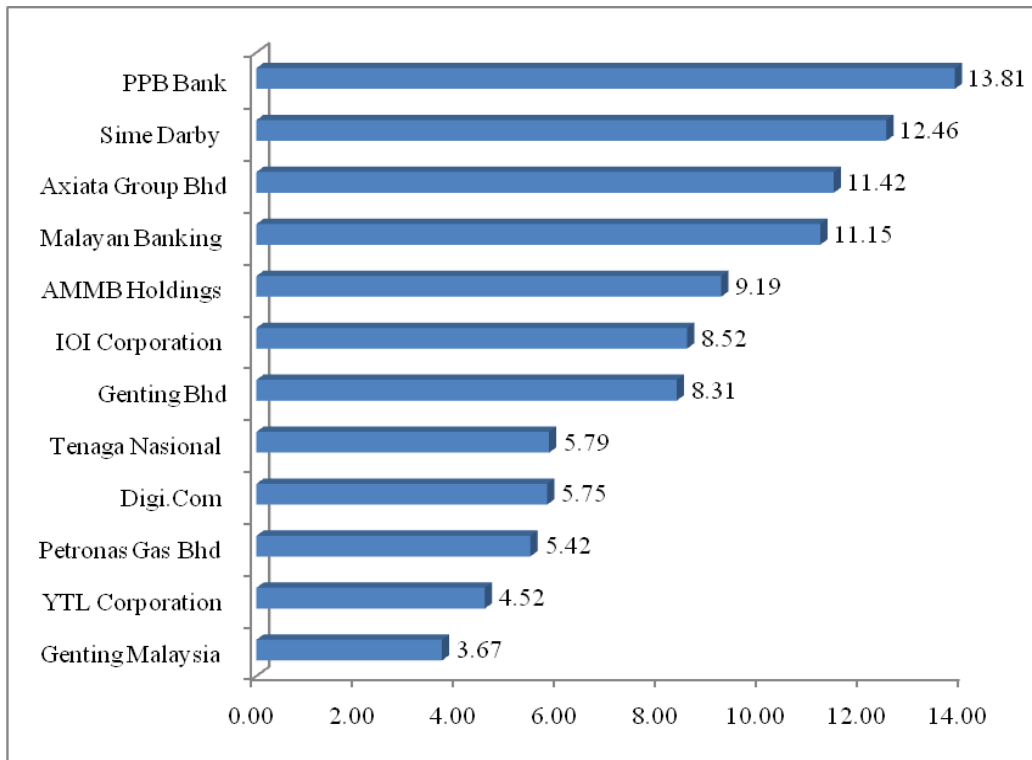


Figure 1: Optimal Portfolio Composition of TEV Model

As shown in Figure 1, based on the investment fund, the optimal portfolio consists of PPB Bank (13.81%), Sime Darby (12.46%), Axiata Group Bhd (11.42%), Malayan Banking (11.15%), AMMB Holdings (9.19%), IOI Corporation (8.52%), Genting Bhd (8.31%), Tenaga Nasional (5.79%), Digi. Com (5.75%), Petronas Gas Bhd (5.42%), YTL Corporation (4.52%) and Genting Malaysia (3.67%). PPB Bank is the most dominant stock in the optimal portfolio whereas Genting Malaysia is the smallest component in the optimal portfolio. Table 2 presents the performance of the optimal portfolio which is constructed by using TEV model.

Table 2: Performance of the Optimal Portfolio

	Number of Stocks	Weekly Mean Return (%)	Tracking Error (%)	Weekly Excess Return (%)
FBMKLCI Index (Benchmark)	30	0.1802	-	-
TEV Model	12	0.2198	0.2960	0.0396

Based on Table 2, the weekly mean return for FBMKLCI Index is 0.1802% based on the study period. The optimal portfolio of TEV model is tracking FBMKLCI Index with weekly mean return 0.2198% at minimum tracking error 0.2960% which approaches zero tracking error. This implies that the optimal portfolio of TEV model is able to track FBMKLCI Index effectively in Malaysia. In addition, the positive deviation from FBMKLCI Index indicates that the optimal portfolio is able to generate weekly excess return 0.0396% over the benchmark index return.

4. Conclusions

This paper discusses about the stock selection in constructing the optimal portfolio to track Malaysia stock market index by using TEV model. The optimal portfolio consists of 12 stocks with different weights to track FBMKLCI Index in Malaysia. The minimum tracking error of the portfolio is 0.2960% which approaches zero tracking error. This implies that the optimal portfolio of TEV model is able to track FBMKLCI Index effectively. Therefore, the TEV model is appropriate for the investors to track the benchmark stock market index in Malaysia. The significance of this study is to determine the optimal portfolio composition in tracking Malaysia stock market index which generates weekly excess return 0.0396% at minimum tracking error 0.2960% with only holding 40% of the index components.

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