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The Validity of Generalized Modal Syllogisms with the Generalized Quantifiers in Square{*most*}

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Abstract

Due to the large number of generalized quantifiers in the English language, this paper only studies the fragment of generalized modal syllogistic that contains the quantifiers in Square{*all*} and Square{*most*}. On the basis of generalized quantifier theory, possible-world semantics, and set theory, this paper shows that there are reducible relations between/among the generalized modal syllogism □EM◇O-3 and at least the other 29 valid generalized modal syllogisms. This method can also be used to study syllogisms with other generalized quantifiers. The results obtained by means of formal deductive method have not only consistency, but also theoretical value for the development of inference theory in artificial intelligence.

Key words: generalized modal syllogisms; reducibility; modality; validity

1. Introduction

Syllogism is one of the significant forms of reasoning in natural language and human thinking. There are various kinds of syllogisms, such as Aristotelian syllogisms (Patzig, 1969; Long, 2023; Hui, 2023), Aristotelian modal syllogisms (Johnson, 2004; Łukasiewicz, 1957; Cheng and Xiaojun, 2023), generalized syllogisms (Murinová and Novák, 2012; Xiaojun and Baoxiang, 2021; Endrullis and Moss, 2015), and generalized modal syllogisms (Jing and Xiaojun, 2023).

Although many generalized modal syllogisms exist in natural language, there is little literature on their reducibilities. Therefore, this paper mainly focuses on them. The four Aristotelian quantifiers (that is, *not all*, *all*, *some* and *no*) constitute Square{*all*}. And '*most*' and its three negative (i.e. inner, outer and dual), *fewer than half of the*, *at most half of the*, *and at least half of the*, form Square{*most*}. The generalized modal syllogisms studied in this paper only involve the quantifiers in Square{*all*} and Square{*most*}.

2. Preliminaries

In this paper, let w, v and z be the lexical variables, which are elements in the set W, V and Z respectively, D be the domain of lexical variables, $|W|$ the cardinality of the set W, and m, n, s and t propositional variables. Q stands for any generalized quantifiers, $\neg Q$ and $Q\neg$ for the outer and inner negative quantifier of Q respectively. The generalized modal syllogisms discussed in this paper comprise the following sentences as follows: 'all *w*s are *v*s', 'no *w*s are *v*s', 'some *w*s are *v*s', 'not all *w*s are *v*s', 'most *w*s are *v*s', 'fewer than half of the *w*s are *v*s', 'at most half of the *w*s are *v*s', and 'at least half of the *w*s are *v*s'. They can be denoted as: *all*(w, v), *no*(w, v), *some*(w, v), *not all*(w, v), *most*(w, v), *fewer than half of the*(w, v), *at most half of the*(w, v), *at least half of the*(w, v), and are respectively abbreviated as Proposition A, E, I, O, M, F, H and S.

A non-trivial generalized modal syllogism includes at least one and at most three non-overlapping modalities (possible modality ($\Diamond$) or necessary modality ($\Box$)) and non-trivial generalized quantifiers, such as the quantifiers in Square{*most*}.

Example 1:

Major premise: No grapes are necessarily blueberries.

Minor premise: Most grapes are purple fruits.

Conclusion: Not all purple fruits are possibly blueberries.

Let w be the lexical variable for a blueberry in the domain, v be the lexical variable for a grape in the domain, and z be the lexical variable for a purple fruit in the domain. Then the syllogism in example 1 can be formalized as: $\Box$*no*(v, w)$\wedge$*most*(v, z)$\rightarrow\Diamond$*not all*(z, w), which abbreviated as $\Box$EM$\Diamond$O-3.

According to generalized quantifier theory, set theory (Halmos, 1974) and possible world semantics (Chellas, 1980), the truth value definitions of sentences with quantification, relevant facts and rules used in the paper are as follows:

Definition 1 (truth value definitions):

(1.1) *all*(w, v) is true when and only when $W\subseteq V$ is true in all real worlds.

(1.2) *no*(w, v) is true when and only when $W\cap V=\varnothing$ is true in all real worlds.

(1.3) *some*(w, v) is true when and only when $W\cap V\neq\varnothing$ is true in all real worlds.

(1.4) *not all*(w, v) is true when and only when $W\not\subseteq V$ is true in all real worlds.

(1.5) *most*(w, v) is true when and only when $|W\cap V|>0.5|W|$ is true in all real worlds.

(1.6) $\Box$*all*(w, v) is true when and only when $W\subseteq V$ is true in all possible worlds.

(1.7) $\Diamond$*all*(w, v) is true when and only when $W\subseteq V$ is true in some possible worlds.

(1.8) $\Box$*no*(w, v) is true when and only when $W\cap V=\varnothing$ is true in all possible worlds.

(1.9) $\Diamond$*no*(w, v) is true when and only when $W\cap V=\varnothing$ is true in some possible worlds.

(1.10) $\Box$*some*(w, v) is true when and only when $W\cap V\neq\varnothing$ is true in all possible worlds.

(1.11) $\Diamond$*some*(w, v) is true when and only when $W\cap V\neq\varnothing$ is true in some possible worlds.

(1.12) $\Box$*not all*(w, v) is true when and only when $W\not\subseteq V$ is true in all possible worlds.

(1.13) $\Diamond$*not all*(w, v) is true when and only when $W\not\subseteq V$ is true in some possible worlds.

(1.14) $\Box$*most*(w, v) is true when and only when $|W\cap V|>0.5|W|$ is true in all possible worlds.

(1.15) $\Diamond$*most*(w, v) is true when and only when $|W\cap V|>0.5|W|$ is true in some possible worlds.

Definition 2 (inner negation): $Q\neg(w, v)=_{\text{def}}Q(w, D\text{-}v)$.

Definition 3 (outer negation): $\neg Q(w, v)=_{\text{def}}$It is not that $Q(w, v)$.

Fact 1 (inner negation):

(1.1) $\vdash$*all*$(w, v)\leftrightarrow$*no*$\neg(w, v)$;

(1.2) $\vdash$*no*$(w, v)\leftrightarrow$*all*$\neg(w, v)$;

(1.3) $\vdash$*some*$(w, v)\leftrightarrow$*not all*$\neg(w, v)$;

(1.4) $\vdash$*not all*$(w, v)\leftrightarrow$*some*$\neg(w, v)$;

(1.5) $\vdash$*fewer than half of the*$(w, v)\leftrightarrow$*most*$\neg(w, v)$;

(1.6) $\vdash$*most*$(w, v)\leftrightarrow$*fewer than half of the*$\neg(w, v)$;

(1.7) $\vdash$*at most half of the*$(w, v)\leftrightarrow$*at least half of the*$\neg(w, v)$;

(1.8) $\vdash$*at least half of the*$(w, v)\leftrightarrow$*at most half of the*$\neg(w, v)$.

Fact 2 (outer negation):

(2.1) $\vdash\neg$*not all*$(w, v)\leftrightarrow$*all*(w, v);

(2.2) $\vdash\neg$*all*$(w, v)\leftrightarrow$*not all*(w, v);

(2.3) $\vdash\neg$*no*$(w, v)\leftrightarrow$*some*(w, v);

(2.4) $\vdash\neg$*some*$(w, v)\leftrightarrow$*no*(w, v);

(2.5) $\vdash\neg$*most*$(w, v)\leftrightarrow$*at most half of the*(w, v);

(2.6) $\vdash\neg$*at most half of the*$(w, v)\leftrightarrow$*most*(w, v);

(2.7) $\vdash\neg$*fewer than half of the*$(w, v)\leftrightarrow$*at least half of the*(w, v);

(2.8) $\vdash\neg$*at least half of the*$(w, v)\leftrightarrow$*fewer than half of the*(w, v).

Fact 3 (dual):

(3.1) $\vdash\neg\Box Q(w, v)\leftrightarrow\Diamond\neg Q(w, v)$;

(3.2) $\vdash\neg\Diamond Q(w, v)\leftrightarrow\Box\neg Q(w, v)$.

Fact 4 (symmetry):

(4.1) $\vdash$*some*$(w, v)\leftrightarrow$*some*(v, w);

(4.2) $\vdash$*no*$(w, v)\leftrightarrow$*no*(v, w).

Fact 5 (subordination):

(5.1) $\vdash\Box Q(w, v)\rightarrow Q(w, v)$;

(5.2) $\vdash\Box Q(w, v)\rightarrow\Diamond Q(w, v)$;

(5.3) $\vdash Q(w, v) \rightarrow \Diamond Q(w, v)$;

(5.4) $\vdash all(w, v) \rightarrow some(w, v)$;

(5.5) $\vdash no(w, v) \rightarrow not\ all(w, v)$.

Rule 1 (subsequent weakening): If $\vdash(m \wedge n \rightarrow s)$ and $\vdash(s \rightarrow t)$, then $\vdash(m \wedge n \rightarrow t)$.

Rule 2 (anti-syllogism): If $\vdash(m \wedge n \rightarrow s)$, then $\vdash(\neg s \wedge m \rightarrow \neg n)$ or $\vdash(\neg s \wedge n \rightarrow \neg m)$.

3. The Validity of the Syllogism □EM◇O-3

In order to discuss the reducibility of generalized modal syllogisms based on the syllogism □EM◇O-3, it is necessary to prove the validity of the syllogism □EM◇O-3.

Theorem 1 (□EM◇O-3): The generalized modal syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ is valid.

Proof: According to Example 1, □EM◇O-3 is the abbreviation of the syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$. Suppose that $\Box no(v, w)$ and $most(v, z)$ are true, then in virtue of

Definition (1.8), $\Box no(v, w)$ is true when and only when $V \cap W = \varnothing$ is true in all possible worlds. Similarly, in line with Definition (1.5), $most(v, z)$ is true when and only when $|V \cap Z| > 0.5|V|$ is true in all real worlds. Real worlds are elements in the set of all possible worlds. Thus, it is easily seen that $V \cap W = \varnothing$ and $|V \cap Z| > 0.5|V|$ are true in some possible worlds. Then, it is clear that $Z \not\subseteq W$ is true in some possible worlds. $\Diamond not\ all(z, w)$ is true in terms of Definition (1.13). The above proves that the syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ is valid.

4. The Other Generalized Modal Syllogisms Derived from □EM◇O-3

Theorem 1 states that □EM◇O-3 is valid, and '□EM◇O-3→□EM◇O-4' in Theorem 2(1) expresses that the validity of syllogism □EM◇O-4 is deduced from that of syllogism □EM◇O-3. That is to show that there are reducible relations between these two syllogisms, and the others are similar.

Theorem 2: There are at least the following 29 valid generalized modal syllogisms obtained from □EM◇O-3:

(1) □EM◇O-3→□EM◇O-4

(2) □EM◇O-3→□A□EH-2

(3) □EM◇O-3→□AM◇I-1

(4) □EM◇O-3→□AM◇I-3

(5) □EM◇O-3→□EM◇O-4→□A□EH-4

(6) □EM◇O-3→□EM◇O-4→M□A◇I-4

(7) □EM◇O-3→□A□EH-2→□E□AH-2

(8) □EM◇O-3→□AM◇I-1→□EM◇O-1

(9) □EM◇O-3→□AM◇I-3→M□A◇I-3

(10) □EM◇O-3→□A□EH-2→□E□AH-2→□E□AH-1

(11) □EM◇O-3→□AM◇I-1→□EM◇O-1→□EM◇O-2

(12) □EM◇O-3→□AM◇I-3→M□A◇I-3→F□A◇O-3

(13) □EM◇O-3→□A□EH-2→□E□AH-2→□E□AH-1→□A□AS-1

(14) □EM◇O-3→□AM◇I-1→□EM◇O-1→□EM◇O-2→□AF◇O-2

(15) □EM◇O-3→□A□EH-2→□A□E◇H-2

(16) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□E◇H-4

(17) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□M◇I-1

(18) □EM◇O-3→□A□EH-2→□A□E◇H-2→□E□M◇O-3

(19) □EM◇O-3→□A□EH-2→□A□E◇H-2→□E□A◇H-2

(20) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□E◇H-4→□M□A◇I-4

(21) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□E◇H-4→□E□M◇O-4

(22) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□M◇I-1→□E□M◇O-1

(23) □EM◇O-3→□A□EH-2→□A□E◇H-2→□E□M◇O-3→□A□M◇I-3

(24) □EM◇O-3→□A□EH-2→□A□E◇H-2→□E□A◇H-2→□E□A◇H-1

(25) □EM◇O-3→□A□EH-2→□A□E◇H-2→□A□M◇I-1→□E□M◇O-1→□E□M◇O-2

(26) □EM◇O-3→□A□EH-2→□A□E◇H-2→□E□M◇O-3→□A□M◇I-3→□M□A◇I-3

(27) □EM◇O-3→□A□EH-2→□E□AH-2→□E□AH-1→□A□AS-1→□A□A◇S-1

(28) □EM◇O-3→□A□EH-2→□E□AH-2→□E□AH-1→□A□AS-1→□A□A◇S-1

→□A□F◇O-2

(29) □EM◇O-3→□A□EH-2→□E□AH-2→□E□AH-1→□A□AS-1→□A□A◇S-1

→□F□A◇O-3

Proof:

[1] ⊢□*no*(*v*, *w*)∧*most*(*v*, *z*)→◇*not all*(*z*, *w*) (i.e. □EM◇O-3, Theorem 1)

[2] ⊢□*no*(*w*, *v*)∧*most*(*v*, *z*)→◇*not all*(*z*, *w*) (i.e. □EM◇O-4, by [1] and Fact (4.2))

[3] ⊢¬◇*not all*(*z*, *w*)∧□*no*(*v*, *w*)→¬*most*(*v*, *z*) (by [1] and Rule 2)

[4] ⊢□¬*not all*(*z*, *w*)∧□*no*(*v*, *w*)→¬*most*(*v*, *z*) (by [3] and Fact (3.2))

[5] ⊢□*all*(*z*, *w*)∧□*no*(*v*, *w*)→*at most half of the*(*v*, *z*)

(i.e. □A□EH-2, by [4], Fact (2.1) and Fact (2.5))

[6] ⊢¬◇*not all*(*z*, *w*)∧*most*(*v*, *z*)→¬□*no*(*v*, *w*) (by [1] and Rule 2)

[7] ⊢□¬*not all*(*z*, *w*)∧*most*(*v*, *z*)→◇¬*no*(*v*, *w*) (by [6], Fact (3.1) and Fact (3.2))

[8] ⊢□*all*(*z*, *w*)∧*most*(*v*, *z*)→◇*some*(*v*, *w*) (i.e. □AM◇I-1, by [7], Fact (2.1) and Fact (2.3))

[9] ⊢□*all*¬(*v*, *w*)∧*most*(*v*, *z*)→◇*some*¬(*z*, *w*) (by [1], Fact (1.2) and Fact (1.4))

[10] ⊢□*all*(*v*, *D*-*w*)∧*most*(*v*, *z*)→◇*some*(*z*, *D*-*w*) (i.e. □AM◇I-3, by [9] and Definition 2)

[11] ⊢¬◇*not all*(*z*, *w*)∧□*no*(*w*, *v*)→¬*most*(*v*, *z*) (by [2] and Rule 2)

[12] ⊢□¬*not all*(*z*, *w*)∧□*no*(*w*, *v*)→¬*most*(*v*, *z*) (by [11] and Fact (3.2))

[13] ⊢□*all*(*z*, *w*)∧□*no*(*w*, *v*)→*at most half of the*(*v*, *z*)

(i.e. □A□EH-4, by [12], Fact (2.1) and Fact (2.5))

[14] ⊢¬◇*not all*(*z*, *w*)∧*most*(*v*, *z*)→¬□*no*(*w*, *v*) (by [2] and Rule 2)

[15] ⊢□¬*not all*(*z*, *w*)∧*most*(*v*, *z*)→◇¬*no*(*w*, *v*) (by [14], Fact (3.1) and Fact (3.2))

[16] ⊢□*all*(*z*, *w*)∧*most*(*v*, *z*)→◇*some*(*w*, *v*)

(i.e. M□A◇I-4, by [15], Fact (2.1) and Fact (2.3))

[17] ⊢□*no*¬(*z*, *w*)∧□*all*¬(*v*, *w*)→*at most half of the*(*v*, *z*) (by [5], Fact (1.1) and Fact (1.2))

[18] ⊢□*no*(*z*, *D*-*w*)∧□*all*(*v*, *D*-*w*)→*at most half of the*(*v*, *z*)

(i.e. □E□AH-2, by [17] and Definition 2)

[19] ⊢□*no*¬(*z*, *w*)∧*most*(*v*, *z*)→◇*not all*¬(*v*, *w*) (by [8], Fact (1.1) and Fact (1.3))

[20] ⊢□*no*(*z*, *D*-*w*)∧*most*(*v*, *z*)→◇*not all*(*v*, *D*-*w*) (i.e. □EM◇O-1, by [19] and Definition 2)

[21] ⊢□*all*(*v*, *D*-*w*)∧*most*(*v*, *z*)→◇*some*(*D*-*w*, *z*) (i.e. M□A◇I-3, by [10] and Fact (4.1))

[22] ⊢□*no*(*D*-*w*, *z*)∧□*all*(*v*, *D*-*w*)→*at most half of the*(*v*, *z*)

(i.e. □E□AH-1, by [18] and Fact (4.2))

[23] ⊢□*no*(*D*-*w*, *z*)∧*most*(*v*, *z*)→◇*not all*(*v*, *D*-*w*) (i.e. □EM◇O-2, by [20] and Fact (4.2))

[24] ⊢□*all*(*v*, *D*-*w*)∧*fewer than half of the*¬(*v*, *z*)→◇*not all*¬(*D*-*w*, *z*)

(by [21], Fact (1.6) and Fact (1.3))

[25] ⊢□*all*(*v*, *D*-*w*)∧*fewer than half of the*(*v*, *D*-*z*)→◇*not all*(*D*-*w*, *D*-*z*)

(i.e. F□A◇O-3, by [24] and Definition 2)

[26] ⊢□*all*¬(*D*-*w*, *z*)∧□*all*(*v*, *D*-*w*)→*at least half of the*¬(*v*, *z*)

(by [22], Fact (1.2) and Fact (1.7))

[27] ⊢□*all*(*D*-*w*, *D*-*z*)∧□*all*(*v*, *D*-*w*)→*at least half of the*(*v*, *D*-*z*)

(i.e. □A□AS-1, by [26] and Definition 2)

[28] ⊢□*all*¬(*D*-*w*, *z*)∧*fewer than half of the*¬(*v*, *z*)→◇*not all*(*v*, *D*-*w*)

(by [23], Fact (1.2) and Fact (1.6))

[29] ⊢□*all*(*D*-*w*, *D*-*z*)∧*fewer than half of the*(*v*, *D*-*z*)→◇*not all*(*v*, *D*-*w*)

(i.e. □AF◇O-2, by [28] and Definition 2)

[30] ⊢□*all*(*z*, *w*)∧□*no*(*v*, *w*)→◇*at most half of the*(*v*, *z*)

(i.e. □A□E◇H-2, by [5], Fact (5.3) and Rule 1)

[31] ⊢□*all*(*z*, *w*)∧□*no*(*w*, *v*)→◇*at most half of the*(*v*, *z*)

(i.e. □A□E◇H-4, by [30] and Fact (4.2))

[32] ⊢¬◇*at most half of the*(v, z)∧□*all*(z, w)→¬□*no*(v, w) (by [30] and Rule 2)

[33] ⊢□¬*at most half of the*(v, z)∧□*all*(z, w)→◇¬*no*(v, w) (by [32], Fact (3.1) and Fact (3.2))

[34] ⊢□*most*(v, z)∧□*all*(z, w)→◇*some*(v, w)

(i.e. □A□M◇I-1, by [33], Fact (2.6) and Fact (2.3))

[35] ⊢¬◇*at most half of the*(v, z)∧□*no*(v, w)→¬□*all*(z, w) (by [30] and Rule 2)

[36] ⊢□¬*at most half of the*(v, z)∧□*no*(v, w)→◇¬*all*(z, w) (by [35], Fact (3.1) and Fact (3.2))

[37] ⊢□*most*(v, z)∧□*no*(v, w)→◇*not all*(z, w)

(i.e. □E□M◇O-3, by [36], Fact (2.6) and Fact (2.2))

[38] ⊢□*no*¬(z, w)∧□*all*¬(v, w)→◇*at most half of the*(v, z) (by [30], Fact (1.1) and Fact (1.2))

[39] ⊢□*no*(z, D-w)∧□*all*(v, D-w)→◇*at most half of the*(v, z)

(i.e. □E□A◇H-2, by [38] and Definition 2)

[40] ⊢¬◇*at most half of the*(v, z)∧□*all*(z, w)→¬□*no*(w, v) (by [31] and Rule 2)

[41] ⊢□¬*at most half of the*(v, z)∧□*all*(z, w)→◇¬*no*(w, v) (by [40], Fact (3.1) and Fact (3.2))

[42] ⊢□*most*(v, z)∧□*all*(z, w)→◇*some*(w, v)

(i.e. □M□A◇I-4, by [41], Fact (2.6) and Fact (2.3))

[43] ⊢¬◇*at most half of the*(v, z)∧□*no*(w, v)→¬□*all*(z, w) (by [31] and Rule 2)

[44] ⊢□¬*at most half of the*(v, z)∧□*no*(w, v)→◇¬*all*(z, w) (by [43], Fact (3.1) and Fact (3.2))

[45] ⊢□*most*(v, z)∧□*no*(w, v)→◇*not all*(z, w)

(i.e. □E□M◇O-4, by [44], Fact (2.6) and Fact (2.2))

[46] ⊢□*most*(v, z)∧□*no*¬(z, w)→◇*not all*¬(v, w) (by [34], Fact (1.1) and Fact (1.3))

[47] ⊢□*most*(v, z)∧□*no*(z, D-w)→◇*not all*(v, D-w)

(i.e. □E□M◇O-1, by [46] and Definition 2)

[48] ⊢□*most*(v, z)∧□*all*¬(v, w)→◇*some*¬(z, w) (by [37], Fact (1.2) and Fact (1.4))

[49] ⊢□*most*(v, z)∧□*all*(v, D-w)→◇*some*(z, D-w)

(i.e. □A□M◇I-3, by [48] and Definition 2)

[50] ⊢□*no*(*D*-*w*, *z*)∧□*all*(*v*, *D*-*w*)→◇*at most half of the*(*v*, *z*)

(i.e. □E□A◇H-1, by [39] and Fact (4.2))

[51] ⊢□*most*(*v*, *z*)∧□*no*(*D*-*w*, *z*)→◇*not all*(*v*, *D*-*w*)

(i.e. □E□M◇O-2, by [47] and Fact (4.2))

[52] ⊢□*most*(*v*, *z*)∧□*all*(*v*, *D*-*w*)→◇*some*(*D*-*w*, *z*)

(i.e. □M□A◇I-3, by [49] and Fact (4.1))

[53] ⊢□*all*(*D*-*w*, *D*-*z*)∧□*all*(*v*, *D*-*w*)→◇*at least half of the*(*v*, *D*-*z*)

(i.e. □A□A◇S-1, by [27], Fact (5.3) and Rule 1)

[54] ⊢¬◇*at least half of the*(*v*, *D*-*z*)∧□*all*(*D*-*w*, *D*-*z*)→¬□*all*(*v*, *D*-*w*) (by [53] and Rule 2)

[55] ⊢□¬*at least half of the*(*v*, *D*-*z*)∧□*all*(*D*-*w*, *D*-*z*)→◇¬*all*(*v*, *D*-*w*)

(by [54], Fact (3.1) and Fact (3.2))

[56] ⊢□*fewer than half of the*(*v*, *D*-*z*)∧□*all*(*D*-*w*, *D*-*z*)→◇*not all*(*v*, *D*-*w*)

(i.e. □A□F◇O-2, by [55], Fact (2.8) and Fact (2.2))

[57] ⊢¬◇*at least half of the*(*v*, *D*-*z*)∧□*all*(*v*, *D*-*w*)→¬□*all*(*D*-*w*, *D*-*z*) (by [53] and Rule 2)

[58] ⊢□¬*at least half of the*(*v*, *D*-*z*)∧□*all*(*v*, *D*-*w*)→◇¬*all*(*D*-*w*, *D*-*z*)

(by [57], Fact (3.1) and Fact (3.2))

[59] ⊢□*fewer than half of the*(*v*, *D*-*z*)∧□*all*(*v*, *D*-*w*)→◇*not all*(*D*-*w*, *D*-*z*)

(i.e. □F□A◇O-3, by [58], Fact (2.8) and Fact (2.2))

Now, the other 29 generalized modal syllogisms have been deduced from the validity of □EM◇O-3. Similarly, more valid syllogisms can be inferred from it. This indicates that there are reducible relations between/among these syllogisms. Their validity can be proven similar to Theorem 1.

5. Conclusion

Due to the large number of generalized quantifiers in the English language, this paper only studies the fragment of generalized modal syllogistic that contains the quantifiers in

Square{*all*} and Square{*most*}. This paper proves that there are reducible relations between/ among the generalized modal syllogism □EM◇O-3 and at least the above 29 valid generalized modal syllogisms. To be specific, this paper firstly proves the validity of □EM◇O-3 on the basis of generalized quantifier theory, possible-world semantics, and set theory. Then, according to some facts and inference rules, the above 29 valid generalized modal syllogisms are derived from □EM◇O-3.

This method can also be used to study syllogisms with other generalized quantifiers, such as *at most* 1/3 *of the*, *more than* 1/3 *of the*, *at least* 2/3 *of the*, *fewer than* 2/3 *of the.* It is obvious that the above results obtained by deduction have not only consistency, but also theoretical value for the development of inference theory in artificial intelligence.

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