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The Reductions between/among Aristotelian Syllogisms Based on the Syllogism AII-3

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Abstract

With the help of the definitions of the inner and outer negative Aristotelian quantifiers, the symmetry of Aristotelian quantifiers *no* and *some*, the anti-syllogism rules and the subsequent weakening rule, this paper illustrates the reducible relations between valid Aristotelian syllogisms of different figures and different forms. More specifically, this paper deduces the remaining 23 valid Aristotelian syllogisms only from the valid syllogism AII-3, and establishes a simple and clear formal axiom system for Aristotelian syllogistic. This formal and innovative research is not only beneficial to the study of reducible relations between other types of syllogisms, such as generalized syllogisms and modal syllogisms, but also to the knowledge representation, knowledge reasoning and natural language information processing in artificial intelligence.

Key words: Aristotelian syllogisms; axioms; Aristotelian quantifiers; reductions

1. Introduction

Syllogistic reasoning is one of the common forms of reasoning in natural language and human thinking (Cheng, 2023). There are various types of syllogism, such as Aristotelian syllogisms (Hui, 2022), generalized syllogisms (Endrullis and Moss, 2015), and modal syllogisms (Johnson, 2004; Jing and Xiaojun, 2023). This paper focuses on the reducibility of Aristotelian syllogisms. If the validity of one syllogism can be inferred from the validity of another syllogism, then it is said that there is reducibility between these two syllogisms(Long, 2023) .

A syllogism has two premises, one conclusion, and three lexical variables. Only 24 syllogisms are valid in the 256 Aristotelian syllogisms (Xiaojun et al., 2022). The Aristotelian school claimed that all valid syllogisms can be derived from the two syllogisms AAA-1 and EAE-1 (Westerståhl, 2007). On the basis of generalized quantifier theory, Xiaojun and Sheng (2016) derived the other 22 valid Aristotelian syllogisms from the two syllogisms mentioned above. Making use of reasoning rules of propositional logic and taking the syllogisms AAA-1 and AII-3 as basic axioms, Łukasiewicz (1957) derived the other 22 valid Aristotelian syllogisms. With the help of generalized quantifier theory, Mengyao and Xiaojun (2020) gives a more intuitive and clear illustration of Łukasiewicz's (1957) results. While this paper will explain how to derive the remaining 23 valid syllogisms from just one Aristotelian syllogism on the basis of generalized quantifier theory (Peters and Westerståhl, 2006) and set theory (Halmos, 1974).

2. Relevant Basic Knowledge

The figures of Aristotelian syllogisms are defined as the usual convention (Xiaojun, 2020). In this paper, let Q be any of the four Aristotelian quantifiers *all*, *no*, *some* and *not all.* G, H and K represent lexical variables in syllogisms, p, q and r well-formed formulas. "$\vdash$ " means that a proposition or syllogism can be asserted.

Aristotelian syllogisms contain the four categorical propositions in the following forms: All Gs are H, No Gs are H, Some Gs are H, Not all Gs are H. The four propositions can be respectively symbolized as $all(G, H)$, $no(G, H)$, $some(G, H)$, and $not\ all(G, H)$, and abbreviated as the proposition A, E, I, O respectively. For example, 'All Hs are K, and some Hs are G, then some Gs are K', it is denoted by $all(H, K)\rightarrow(some(H, G)\rightarrow some(G, K))$, and abbreviated as AII-3. The other cases are similar. The following syllogism is an instance of the syllogism AII-3:

Major premise: All dogs are carnivore animals.

Minor premise: Some dogs are white dogs.

Conclusion: Some white dogs are carnivore animals.

Let H be the dogs in the domain, K the carnivore animals in the domain, and G the white dogs in the domain. Then the syllogism can be formalized by $all(H, K)\rightarrow(some(H, G)\rightarrow some(G, K))$, that is, the syllogism AII-3.

3. Aristotelian Syllogistic System

Aristotelian syllogistic system contains the following primitive symbols, formation rules, axioms and reasoning rules, etc.

3.1 Primitive Symbols

(1) lexical variables: G, H, K

(2) unary negative operator: $\neg$

(3) binary implication operator: $\rightarrow$

(4) quantifier: *all*, *some*

(5) brackets: (,)

3.2 Formation Rules

(1) If Q is a quantifier, and G and H are lexical variables, then $Q(G, H)$ is a well-formed formula.

(2) If p and q are well-formed formulas, then $\neg p$ *and* $p\rightarrow q$ are well-formed formulas.

(3) Only the formulas obtained through (1) and (2) are well-formed formulas.

For example, $all(G, H)$, $some(G, H)$, and $all(G, H)\rightarrow\neg all(H, K)$ are well-formed formulas, which read respectively as 'all Gs are H','some Gs are H ' and 'if all Gs are H, then that all Hs are K is false'. Others are similar.

3.3 Related Definitions

(1) Definition of connective $\wedge$: $(p\wedge q) =_{\text{def}} \neg(p\rightarrow\neg q)$.

(2) Definition of connective $\leftrightarrow$: $(p\leftrightarrow q) =_{\text{def}} (p\rightarrow q)\wedge(q\rightarrow p)$.

(3) Definition of inner negative quantifier: $Q\neg(G, H) =_{\text{def}} Q(G, U-H)$.

(4) Definition of outer negative quantifier: $\neg Q(G, H) =_{\text{def}}$ It is not that $Q(G, H)$.

(5) Truth definition of Aristotelian quantifier *all*: $all(G, H) =_{\text{def}} G\subseteq H$.

(6) Truth definition of Aristotelian quantifier *some*: $some(G, H) =_{\text{def}} G\cap H\neq\varnothing$.

(7) Truth definition of Aristotelian quantifier *no*: $no(G, H) =_{\text{def}} G\cap H=\varnothing$.

(8) Truth definition of Aristotelian quantifier *not all*: $not\ all(G, H) =_{\text{def}} G\nsubseteq H$.

3.4 Assertion Axioms

(1) A0: if p is a valid formula in propositional logic, then $\vdash p$.

(2) A1: $\vdash all(G, G)$.

(3) A2: $\vdash some(G, G)$.

(4) A3 (i.e. AII-3): $\vdash$ $all(H, K)\to(some(H, G)\to some(G, K))$

3.5 Reasoning Rules for Assertion

Aristotelian syllogistic is a branch of predicate logic (Cori and Lascar, 2000), and the latter is an extension of classical propositional logic (Hamilton, 1978), thus the theorems and reasoning rules of classical propositional logic as the following are also applicable in Aristotelian syllogistic.

(1) RU (Uniform substitution): if q is obtained from p by uniformly replacing one variable with another, then $\vdash q$ can be derived from $\vdash p$.

(2) MP (Modus Ponens): $\vdash q$ can be deduced from $\vdash (p\to q)$ and $\vdash p$.

(3) RD (Definiens and definiendum interchange): $\vdash (\ldots\beta\ldots)$ can be deduced from $\vdash (\ldots\alpha\ldots)$ and $\alpha=_{def}\beta$, and vice versa.

(4) RE (Substitution of equivalents): From $\vdash (\ldots\alpha\ldots)$ and $\alpha\leftrightarrow\beta$ infer $\vdash (\ldots\beta\ldots)$, and vice versa.

(5) RN (Double negative): $\vdash p$ can be deduced from $\vdash \neg\neg p$, and vice versa.

(6) RA (Antecedent interchange): $\vdash (q\to(p\to r))$ can be deduced from $\vdash (p\to(q\to r))$.

(7) RW(Subsequent weakening): From $\vdash (p\to(q\to r))$ and $\vdash (r\to s)$ infer $\vdash (p\to(q\to s))$.

(8) RR (Reversal rule): $\vdash (\neg q\to\neg p)$ can be deduced from $\vdash (p\to q)$.

(9) RS-1 (Anti-syllogism 1): From $\vdash (p\to(q\to r))$ infer $\vdash (p\to(\neg r\to\neg q))$;

(10) RS-2 (Anti-syllogism 2): From $\vdash (p\to(q\to r))$ infer $\vdash (q\to(\neg r\to\neg p))$.

4. Related Theorems

Generalized quantifiers theory (Xiaojun, 2014) says that: (1) *all* and *no*, *some* and *not all* are inner negative each other, that is, *all*=*no*¬, *no*=*all*¬; *some*=*not all*¬, *not all*= *some*¬ (i.e. the following Theorem 1); (2) *all* and *not all*, *some* and *no* are outer negative each other, that is, *all*=¬*not all*, *not all*=¬*all*; *some*=¬*no*, *no*=¬*some* (i.e. the following Theorem 2).

Theorem 1 (inner negative theorem)

(1) $\vdash$ $all(G, H)\leftrightarrow no\neg(G, H)$; (2) $\vdash$ $no(G, H)\leftrightarrow all\neg(G, H)$;

(3) $\vdash$ $some(G, H)\leftrightarrow not\ all\neg(G, H)$; (4) $\vdash$ $not\ all(G, H)\leftrightarrow some\neg(G, H)$.

Proof. This theorem can be proved by definitions in section 3.3 and rules in section 3.5.

[1] $\vdash$ $all(G, H)\leftrightarrow all\neg\neg(G, H)$ (by RN)

[2] $\vdash$ $no(G, H) =_{def} all\neg(G, H)$ (by the definition of quantifier *no*)

[3] $\vdash$ *all¬¬(G, H)↔no¬(G, H)* (from [1] and [2] by RD)

[4] $\vdash$ *all(G, H)↔no¬(G, H)* (i.e.(1), from [1] and [3] by RE)

Other proofs are similar. ■

Theorem 2 (outer negative theorem)

(1) $\vdash$ *all(G, H)↔¬not all(G, H)*; (2) $\vdash$ *not all(G, H)↔¬all(G, H)*;

(3) $\vdash$ *some(G, H)↔¬no(G, H)*; (4) $\vdash$ *no(G, H)↔¬some(G, H)*.

Proof. This fact can be proved by the definitions in section 3.3.

[1] $\vdash$ *all(G, H)↔¬¬all(G, H)* (by RN)

[2] $\vdash$ *not all(G, H)* $=_{\mathrm{def}}$ *¬all(G, H)* (by the definition of quantifier *not all*)

[3] $\vdash$ *¬¬all(G, H)↔¬not all(G, H)* (from [1] and [2] by RD)

[4] $\vdash$ *all(G, H)↔¬not all(G, H)* (i.e.(1), from [1] and [3] by RE)

Other proofs are similar. ■

In the generalized quantifier theory, Aristotelian quantifiers *some* and *no* have symmetry, that is, they have properties as the following Theorem 3.

Theorem 3 (symmetry theorem):

(1) $\vdash$ *some(G, H)↔some(H, G)*; (2) $\vdash$ *no(G, H)↔no(H, G)*.

Proof.

[1] $\vdash$ *all(H, K)*→(*some(H, G)→some(G, K)*) (by axiom A3)

[2] $\vdash$ *all(H, H)*→(*some(H, G)→some(G, H)*) (from [1] by RU)

[3] $\vdash$ *all(H, H)* (by axiom A1 and RU)

[4] $\vdash$ *some(H, G)→some(G, H)* (from [2] and [3] by MP)

[5] $\vdash$ *some(G, H)→some(H, G)* (i.e. (1), from [4] by RU)

[6] $\vdash$ *some(G, H)↔some(H, G)* (from [4] and [5] by the definition of ↔)

[7] $\vdash$ (*some(H, G)→some(G, H)*)→(*¬some(G, H)→¬some(H, G)*) (by RR)

[8] $\vdash$ *¬some(G, H)→¬some(H, G)* (from [4] and [7] by MP)

[9] $\vdash$ *no(G, H)→no(H, G)* (from [8] by Theorem 2 and RU)

[10] $\vdash$ *no(H, G)→no(G, H)* (from [9] by RU)

[11] $\vdash$ *no(G, H)↔no(H, G)* (i.e. (2), from [9] and [10] by the definition of ↔) ■

Theorem 4 (Assertoric subalternations):

(1) $\vdash$ *all(G, H)→some(G, H)*; (2) $\vdash$ *no(G, H)→not all(G, H)*.

Proof.

[1] $\vdash$ *all(H, K)*→(*some(H, G)→some(G, K)*) (by axiom A3)

[2] $\vdash$ *some(H, G)*→(*all(H, K)→some(G, K)*) (from [1] by RA)

[3] $\vdash$ *some(G, G)*→(*all(G, K)→some(G, K)*) (from [2] by RU)

[4] $\vdash$ *some(G, G)* (by RU and axiom A2)

[5] $\vdash$ *all(G, K)→some(G, K)* (from [3] and [4] by MP)

[6] $\vdash$ *all(G, H)→some(G, H)* (i. e. (1), from [5] by RU)

[7] $\vdash$ (*all(G, H)→some(G, H)*)→(*¬some(G, H)→¬all(G, H)*) (from [6] by RR)

[8] $\vdash$ *¬some(G, H)→¬all(G, H)* (from [6] and [7] by MP)

[9] $\vdash$ *no(G, H)→not all(G, H)* (i. e. (2), from [8] by RE and Theorem 2) ■

Theorem 5(the validity of the syllogism AII-3): the Aristotelian Syllogism *all(H, K)* → (*some(H, G)→some(G, K)*) is valid.

Proof: Suppose that *all(H, K)* and *some(H, G)* are true, then *all(H, K)*$=_{def}H\subseteq K$ and *some(H, G)* $=_{def}H\cap G\neq\varnothing$ are true by means of Definition (5) and (6) in section 3.3, respectively. It can be seen that $H\subseteq K$ and $H\cap G\neq\varnothing$. It follows that $G\cap K\neq\varnothing$. Thus, *some(G, K)* is true according to Definition (6) in section 3.3, just as desired.

5. How to Deduce the Other 23 Valid Aristotelian Syllogisms from the Syllogism AII-3

In the following theorem 6, $\vdash$ AII-3→AII-1 means that the validity of the syllogism AII-1 can be deduced from the validity of the syllogism AII-3. In other words, there are reducible relations between these two syllogisms. The others are similar. The reductions between different syllogisms are crucial to establish the proof system of Aristotelian syllogistic.

Theorem 6 (relations between different Aristotelian syllogisms): The remaining 23 valid Aristotelian syllogisms can be derived from the syllogism AII-3. More specifically:

(1) $\vdash$ AII-3→AII-1

(2) $\vdash$ AII-3→IAI-3

(3) $\vdash$ AII-3→IAI-3→IAI-4

(4) $\vdash$ AII-3→EIO-3

(5) ⊢ AII-3→EIO-3→EIO-4

(6) ⊢ AII-3→EIO-3→EIO-4→EIO-2

(7) ⊢ AII-3→EAE-2

(8) ⊢ AII-3→EIO-1

(9) ⊢ AII-3→AII-1→AEE-2

(10) ⊢ AII-3→IAI-3→EAE-1

(11) ⊢ AII-3→IAI-4→AEE-4

(12) ⊢ AII-3→EAE-2→EAO-2

(13) ⊢ AII-3→AII-1→AEE-2→AEO-2

(14) ⊢ AII-3→IAI-3→EAE-1→EAO-1

(15) ⊢ AII-3→IAI-4→AEE-4→AEO-4

(16) ⊢ AII-3→IAI-3→EAE-1→AAA-1

(17) ⊢ AII-3→IAI-3→EAE-1→AAA-1→AAI-1

(18) ⊢ AII-3→IAI-3→EAE-1→AAA-1→AAI-1→AAI-4

(19) ⊢ AII-3→EIO-3→EIO-4→EIO-2→AOO-2

(20) ⊢ AII-3→IAI-3→OAO-3

(21) ⊢ AII-3→EAE-2→EAO-2→AAI-3

(22) ⊢ AII-3→EAE-2→EAO-2→AAI-3→EAO-3

(23) ⊢ AII-3→EAE-2→EAO-2→AAI-3→EAO-3→EAO-4

Proof.

[1] ⊢ *all(H, K)*→(*some(H, G)*→*some(G, K))* (AII-3, by axiom A3)

[2] ⊢ *some(H, G)*↔*some(G, H)* (by RU and symmetry theorem)

[3] ⊢ *all(H, K)*→(*some(G, H)*→*some(G, K))* (AII-1, from [1] and [2] by RE)

[4] ⊢ *some(G, K)*↔*some(K, G)* (by RU and symmetry theorem)

[5] ⊢ *all(H, K)→(some(H, G)→some(K, G))* (from [1] and [4] by RE)

[6] ⊢ *some(H, G)→(all(H, K)→some(K, G))* (i.e. IAI-3, from [5] by RA)

[7] ⊢ *all(H, K)→(some(G, H)→some(K, G))* (from [2] and [5] by RE)

[8] ⊢ *some(G, H)→(all(H, K)→some(K, G))* (i.e. IAI-4, from [7] by RA)

[9] $\vdash$ $all(H, K)\leftrightarrow no\neg(H, K)$ (by Theorem 1 and RU)

[10] $\vdash$ $some(G, K)\leftrightarrow not\ all\neg(G, K)$ (by Theorem 1 and RU)

[11] $\vdash$ $no\neg(H, K)\rightarrow(some(H, G)\rightarrow not\ all\neg(G, K))$ (from [1], [9] and [10] by RE)

[12] $\vdash$ $no(H, U-K)\rightarrow(some(H, G)\rightarrow not\ all(G, U-K))$

(from [11] by the definition of inner negation)

[13] $\vdash$ $no(H, K)\rightarrow(some(H, G)\rightarrow not\ all(G, K))$ (i.e. EIO-3, from [12] by RU)

[14] $\vdash$ $no(H, K)\leftrightarrow no(K, H)$ (by RU and symmetry theorem)

[15] $\vdash$ $no(K, H)\rightarrow(some(H, G)\rightarrow not\ all(G, K))$ (i.e. EIO-4, from [13] and [14] by RE)

[16] $\vdash$ $no(K, H)\rightarrow(some(G, H)\rightarrow not\ all(G, K))$ (i.e. EIO-2, from [2] and [15] by RE)

[17] $\vdash$ $all(H, K)\rightarrow(\neg some(G, K)\rightarrow\neg some(H, G))$ (from [1] by RS-1)

[18] $\vdash$ $all(H, K)\rightarrow(no(G, K)\rightarrow no(H, G))$ (from [17] by Theorem 2)

[19] $\vdash$ $no(G, K)\rightarrow(all(H, K)\rightarrow no(H, G))$ (i.e. EAE-2, from [18] by RA)

[20] $\vdash$ $some(H, G)\rightarrow(\neg some(G, K)\rightarrow\neg all(H, K))$ (from [1] by RS-2)

[21] $\vdash$ $some(H, G)\rightarrow(no(G, K)\rightarrow not\ all(H, K))$ (from [20] by Theorem 2)

[22] $\vdash$ $no(G, K)\rightarrow(some(H, G)\rightarrow not\ all(H, K))$ (i.e. EIO-1, from [21] by RA)

[23] $\vdash$ $all(H, K)\rightarrow(\neg some(G, K)\rightarrow\neg some(G, H))$ (from [3] by RS-1)

[24] $\vdash$ $all(H, K)\rightarrow(no(G, K)\rightarrow no(G, H))$ (i.e. AEE-2, from [23] by Theorem 2)

[25] $\vdash$ $all(H, K)\rightarrow(\neg some(K, G)\rightarrow\neg some(H, G))$ (from [5] by RS-1)

[26] $\vdash$ $all(H, K)\rightarrow(no(K, G)\rightarrow no(H, G))$ (from [25] by Theorem 2)

[27] $\vdash$ $no(K, G)\rightarrow(all(H, K)\rightarrow no(H, G))$ (i.e. EAE-1, from [26] by RA)

[28] $\vdash$ $all(H, K)\rightarrow(\neg some(K, G)\rightarrow\neg some(G, H))$ (from [7] by RS-1)

[29] $\vdash$ $all(H, K)\rightarrow(no(K, G)\rightarrow no(G, H))$ (i.e. AEE-4, from [28] by Theorem 2)

[30] $\vdash$ $no(H, G)\rightarrow not\ all(H, G)$ (by RU and Theorem 4)

[31] $\vdash$ $all(H, K)\rightarrow(no(G, K)\rightarrow not\ all(H, G))$ (from [30] and [29] by RW)

[32] $\vdash$ $no(G, K)\rightarrow(all(H, K)\rightarrow not\ all(H, G))$ (i.e. EAO-2, from [31] by RA)

[33] $\vdash$ $no(G, H)\rightarrow not\ all(G, H)$ (by Theorem 4)

[34] $\vdash$ $all(H, K)\rightarrow(no(G, K)\rightarrow not\ all(G, H)$ (i.e. AEO-2, from [24] and [33] by RW)

[35] $\vdash$ $all(H, K)\rightarrow(no(K, G)\rightarrow not\ all(H, G))$ (from [27] and [30] by RW)

[36] $\vdash$ *no(K, G)→(all(H, K)→not all(H, G))* (i.e. EAO-1, from [35] by RA)

[37] $\vdash$ *all(H, K)→(no(K, G)→not all(G, H))* (i.e. AEO-4, from [29] and [33] by RW)

[38] $\vdash$ *all¬(K, G)→(all(H, K)→all¬(H, G))*

(similar to [9]-[11], from [27] by Theorem 1 and RU)

[39] $\vdash$ *all(K, G)→(all(H, K)→all(H, G))*

(i.e. AAA-1, similar to [12] and [13], from [38] by Theorem 1 and RU)

[40] $\vdash$ *all(H, G)→some(H, G)* (by RU and Theorem 4)

[41] $\vdash$ *all(K, G)→(all(H, K)→some(H, G))* (i.e. AAI-1, from [39] and [40] by RW)

[42] $\vdash$ *all*(*H*, *K*)→(*all*(*K*, *G*)→*some*(*G*, *H*)) (i.e. AAI-4, from [2] and [41] by RE)

[43] $\vdash$ *all¬(K, H)→(not all¬(G, H)→not all(G, K))*

(similar to [9]-[11], from [16] by Theorem 1 and RU)

[44] $\vdash$ *all(K, H)→(not all(G, H)→not all(G, K))*

(i.e. AOO-2, similar to [12] and [13], from [43] by RU)

[45] $\vdash$ *not all¬(H, G)→(all(H, K)→not all¬(K, G))*

(similar to [9]-[11], from [6] by Theorem 1 and RU)

[46] $\vdash$ *not all(H, G)→(all(H, K)→not all(K, G))*

(i.e. OAO-3, similar to [12] and [13], from [45] by RU)

[47] $\vdash$ *all(H, K)→(¬not all(H, G)→¬no(G, K))* (from [32] by RS-2)

[48] $\vdash$ *all(H, K)→(all(H, G)→some(G, K))* (i.e. AAI-3, from [47] by Theorem 2 and RE)

[49] $\vdash$ *no¬(H, K)→(all(H, G)→not all¬(G, K))*

(similar to [9]-[11], from [48] by Theorem 1 and RU)

[50] $\vdash$ *no(H, K)→(all(H, G)→not all(G, K))*

(i.e. EAO-3, similar to [12] and [13], from [49] by RU)

[51] $\vdash$ *no(K, H)→(all(H, G)→not all(G, K))* (i.e. EAO-4, from [14] and [50] by RE) ■

So far, on the basis of the above definitions, rules and theorems, Theorem 6 derives the remaining 23 valid Aristotelian syllogisms only from the valid syllogism AII-3. In other words, there are reducible relations between this syllogism AII-3 and the other 23 valid syllogisms.

6. Conclusion

The main work and conclusions of this paper are as follows: (1) By using the symmetry of Aristotelian quantifiers *no* and *some*, the definitions of the inner and outer negative Aristotelian quantifiers, the anti-syllogism rules and the subsequent weakening rule, etc., this paper illustrates the reducible relations between valid syllogisms of different figures and different forms. (2) This paper derives the remaining 23 valid syllogisms just from the valid syllogism AII-3 on the basis of generalized quantifier theory. And then a minimalist formal axiom system can be established for Aristotelian syllogistic. (3) The reducible relations between syllogisms of different figures and different forms exemplify the dialectical materialist world view in which things are universally connected.

This study provides a unified mathematical research paradigm for the study of reducible relations between other types of syllogisms, such as generalized syllogisms and modal syllogisms. How to use the research method of this paper to study the reducible relations between other types of syllogisms. This problem needs to be further studied.

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