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Lorentz Transformations for Two Frames of Reference

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Annotation. It is shown that the Lorentz transformations for two reference frames are Galilean transformations, additionally taking into account the initial displacement of the reference frames and the initial difference in the readings of clocks located in these frames. It is established that the disynchronization in the clock readings does not affect the recalculation of coordinates at the transition from one system to another. It is shown that the transformations for two reference frames obey the Rule of Simultaneity, in which the parameters of the speed of light are absent because they are unnecessary. Transformations for two systems are applicable both for inertial reference systems and for non-inertial ones irrespective of the type, velocity and direction of their mutual motion.

Keywords. Lorentz transformations, Galileo transformations, special theory of relativity.

Introduction

The general Lorentz coordinate transformations derived from the laws of classical mechanics [3], [4] have the following record:

$$x' = \frac{x - vt}{k_S} \qquad x = \frac{x' + vt'}{k'_S}$$

$$t' = \frac{t - \beta x/V}{k_S} \qquad t = \frac{t' + \beta x'/V}{k'_S} \tag{1}$$

where: $k_S\, k'_S = 1 - \beta^2$

$\beta = v/V$

v – velocity between reference frames (K) and (K');

V – velocity between reference frames (K_0) and (K).

The known Lorentz transformations represent a special case in which $(V = c)$ and $(k_S = k'_S)$.

The new conclusion Lorentz transformations reveals their physical meaning and indicates, in particular, that equations (1) operate with three inertial reference systems: (K), (K') and (K_0), of which the last one is privileged (absolute) for the given group of systems under study.

It follows from this that the existing method of deriving Lorentz transformations in the special theory of relativity (STR) contains a fundamental error, since coordinate transformations (1) are considered on a model with two inertial reference systems [1], [2].

As a result, in the group of three reference systems, the third privileged system (K_0) in the STR was mistakenly excluded from the derivation of transformations, and with it the physical meaning of this system (K_0) was deleted from science, i.e. the rejection of the luminiferous medium (ether) was declared.

Ignoring the third frame of reference, Einstein had to resort to ambiguous postulates and translate conventional measures of duration and dimension into a material entity, which eventually led him to misinterpret the properties of Lorentz transformations.

The inconsistency of deriving transformations using postulates is clearly seen in the Lorentz transformations, applied not to three, but to two frames of reference where the speed of the light wave (V) is missing.

Lorentz transformations for two frames of reference

Unlike the Galilean transformations, the Lorentz transformations (1) take into account the initial displacement of the reference frames $(\Delta x' = x' - x)$ and the displacement of the time scale in these systems $(\Delta t' = t' - t)$, which are compensated by the coefficients (k_S, k'_S).

The simultaneity coefficient (k_S) defines the initial coordinate (x') at $(t = 0)$ as $(x' = x/k_S)$. The reverse order is also possible, when the coefficient (k_S) is determined through a given offset $(\Delta x')$:

$$k_S = \frac{x}{x + \Delta x'} \tag{2}$$

A similar transformation is observed with parameters of time. In one case, the velocity (V) determines the initial difference of the clock readings $(\Delta t')$, which is adjusted by the amount of the offset of the reference frames $(\Delta x')$ through (k_S).

$$\Delta t' = \frac{v\,x}{k_S V^2} \tag{3}$$

In another case, other parameters of the equations, including (V), are determined from the already existing difference in clock readings $(\Delta t')$ in different reference systems. And since the values $(v, x, \Delta x')$ are set initially, it turns out that the velocity parameter (V) acts as a matching coefficient. Given that the movement between systems ($K_0 - K$) is not considered, the parameter (V) can be excluded, and its function as a matching element can be expressed through other arguments.

(6)

From (3) we determine the intermediate matching coefficient (V^2).

$$V^2 = \frac{v\,x}{k_S\,\Delta t'} \tag{4}$$

Then the Rule of simultaneity ($k_S\,k'_S = 1 - v^2/V^2$), applied for three reference frames, for two systems taking into account (2) and (4) will have the following form:

$$\boxed{k_S\,k_S{}' = 1 - \frac{v\,\Delta t'}{x + \Delta x'}} \tag{5}$$

From (1), and considering (4), we find the Lorentz transformations for two reference frames, which will have the following record:

$$\boxed{\begin{array}{ll} x' = \dfrac{x - v\,t}{k_S} & x = \dfrac{x' + v\,t'}{k_S{}'} \\[2em] t' = \dfrac{t}{k_S} - \Delta t' & t = \dfrac{t' + \dfrac{x'\Delta t'}{x + \Delta x'}}{k_S{}'} \end{array}}$$

where: $\quad k_S\,k_S{}' = 1 - \dfrac{v\,\Delta t'}{x + \Delta x'}$

As we can see, an observer located in the reference frame (K) is able to calculate the time of future events not only by his own clock (t), but also by the clock of the opposite system (K'), which initially have different readings with a difference of ($\Delta t'$).

The second observer, who is in his own frame of reference (K'), is also able to determine by his watch (t') the moment of the same event, as well as calculate the time (t) by the clock for the first observer, who is in a fixed frame of reference (K).

Here it is necessary to make some clarifications regarding the parameter ($\Delta t'$).

So, in the STR, concepts such as «clock readings» and «synchronicity of their movement» are not shared and are considered identical. This is certainly a mistake, since identical clocks running synchronously have the same periods of oscillation of their pendulums ($T = T'$). But their readings may be different ($\Delta t' = t' - t$) and this difference in readings will always be constant. An example is the time zone, when readings in neighboring time zones differ by one hour, but all clocks in all time zones run synchronously ($t_{UTC-1} < t_{UTC+0} < t_{UTC+1}$).

In turn, the non-synchronicity of the same clock means that the periods of oscillation of their pendulums do not match ($T \neq T'$), although their readings during the day may sometimes coincide. In this case, the readings of such a clock will be related as ($t' = k_t t$), where: ($k_t = T/T'$).

As you can see, it doesn't matter what discrepancy there is in the clock readings ($\Delta t'$)

– it is important to know the initial parameters of these discrepancies: either $(\Delta t_0{}' = t' - t)$, or $(k_t = T/T')$, or both at the same time, and thereby establish the relationship:

$$t' = t(1 \pm k_t) \pm \Delta t_0{}'$$

This is the uniqueness of the mathematical apparatus of Lorentz transformations (6). Even with non–synchronously running clocks, transformations will always be performed. This is explained by the fact that the change in the parameter $(\Delta t')$ is compensated by corresponding changes in the coefficients of simultaneity $(k_S, k_S{}')$ according to the Rule of simultaneity (5).

Consequences of Lorentz transformations for two frames of reference

Using the example of coordinate transformations for two reference systems (6), the direct connection of Lorentz transformations with Galileo transformations is clearly visible.

So, if the clock readings are the same and the clock runs synchronously $(\Delta t' = 0)$, and the coefficient $(k_S = 1)$, i.e. when the axes (X) and (X') are parallel, then the Lorentz transformation (6) acquires the classical notation of Galilean transformations.

In this case, the speed (V), and for a special case it is the speed of light (c), is not involved in the calculations, which is very important. It follows from this that the transition from Lorentz transformations to Galileo transformations does not depend on the value (ratio) of any parameters, but on the presence of these parameters, and therefore the principle of correspondence in STR $(v << c)$ is completely untenable.

Consider the following example, when $(\Delta x' \neq 0)$ and clocks in different reference frames run synchronously $(T = T')$. Then the Lorentz transformations for the two reference frames are simplified and take the following form:

$$x' = \frac{x - vt}{k_S} \qquad x = \frac{x' + vt'}{k_S{}'}$$
$$t' = \frac{t}{k_S} \qquad t = \frac{t'}{k_S{}'} \tag{7}$$

where: $k_S\, k_S{}' = 1$

$k_S = x/(x + \Delta x')$

For this case, the motion of the reference frame (K') can be considered for clarity as motion at an angle (α) to the axis (X) of the system (K), which is defined as $(\cos\alpha = k_S)$. In fact, the motion of the system (K') to the point (x) can be at any (polar) angle.

Next, we consider two variants of the applicability of transformations (7) when the motion is considered in the same time frame, i.e. at $(t = t')$. In this case, the relative (coordinate) velocity appears in the calculations.

The first option. For an observer in the system (K), the velocity of the system (K') will be coordinate and equal to (v), and for the second observer in his system, it will be real (v'), which is defined as:

$$v' = v/k_S$$

where: $k_S = \cos\alpha$

In the case ofa change in velocity [$v = f(t)$], the transformations retain their properties. This means that transformations (6) are applicable to non-inertial reference frames.

2. The second option. The velocities of each system (K) and (K') relative to (K_0) are valid and equal to (v) and (v'). According to the requirement of simultaneity, the velocity (v') will be determined in a similar way, based only on the ratio of coordinates at the initial moment. Let's consider the second case in more detail.

Recall that the Lorentz transformation (1) uses mathematical formalism, when in calculations the actual velocity (v') is replaced by the coordinate time (t'), following the principle:

$$\frac{x}{x'} = \frac{vt}{v't} = \frac{vt}{vt'}$$

Let's use this mathematical technique in the reverse order, i.e. instead of the invariant of the derivative of displacement, i.e. of velocity, we will give the invariant of duration, i.e. of time. And then the Lorentz transformations (7) for each variant is transformed into a new record:

$$x' = \frac{x - vt}{k_S} \qquad x = \frac{x' + v't'}{k_S'} \tag{8}$$

$$t' = t \qquad t = t'$$

where: $k_S\, k_S' = 1$

$k_S = x/(x + \Delta x')$ or $k_S = v/v$,

As we can see, the obtained equations (8) and equations (1) in the direct transformation for coordinates are completely identical. But the equations for the inverse transformation differ in the choice of an invariant. In transformation (8), the invariant is time (t), and in transformation (1) it is velocity (v).

It follows from this that endowing any speed, including the speed of light (c), with the property of invariance, i.e., giving the speed the property of constancy in any frame of reference, is just a choice of one methodology or another in determining the relative position of different frames of reference in space.

Using the Rule of simultaneity, transformations (8) can be simplified by replacing the actual velocity (v') in the system (K') with the velocity (v) in the system (K).

$$x' = \frac{x - vt}{k_S} \qquad x = \frac{x'}{k_S'} + vt' \tag{9}$$

$$t' = t \qquad t = t'$$

where: $k_S\, k_S' = 1$

$k_S = x/(x + \Delta x')$

Transformations (9) are notable for the fact that their feasibility is based on the condition of absolute simultaneity, i.e. the condition of simultaneous arrival of two objects at the same point (x). And although the velocity of the second object (v') is formally absent, it is nevertheless taken into account through the coefficients (k_S) and (k_S').

As you can see, another methodology is used for the reverse transformation, in which two parameters – time (t) and speed (v) – are invariant. In this case, the transformations of coordinates (8) and (9) are completely identical.

This circumstance once again confirms that the transfer of an invariant at the will of the observer from one parameter to another cannot be the reason for the appearance of new physical laws.

Discussion

The properties of Lorentz transformations in (6) are important because they clarify the understanding of the simultaneity of events.

Simultaneity can be judged by various signs. It is possible to observe the simultaneous arrival of two objects at the point of intersection of their trajectories. It is possible to compare the time in receiving a signal, which always depends on the transmission rate of this signal (light, sound, mail) and the location of the observer relative to these events. There are other ways.

In the special theory of relativity, it is generally assumed that two events will be simultaneous if the readings of the clocks located at the points where the events occurred coincide. This approach has led to the fact that the judgment of simultaneity has shifted from the procedure of comparing the signal transit time ($\Delta t \leftrightarrow \Delta t'$) to comparing the clock readings ($t \leftrightarrow t'$), which is not the same thing.

This is the whole essence of Einstein's relativity of simultaneity, when the fact of events that have occurred is replaced by the fact that the observer receives information about these events..

Meanwhile, the opposite pattern is observed in the Lorentz transformation (6): the simultaneity of events is determined by the duration and only then, taking into account the discrepancy ($\Delta t'$), for each clock, their readings are determined at which these events occurred simultaneously. In other words, in transformation (6), the clock readings in the system (K') are calculated taking into account their desynchronization.

Therefore, the question whether such clocks are running correctly or not becomes optional (irrelevant), since the parameters of the desynchronization of the second clock in the transformation are automatically compensated in accordance with Rule (5). That is why the second observer in the reference frame (K') can always correctly determine the moment of simultaneity of events by his incorrectly running clock.

Thus, it can be argued that in Lorentz transformations, the simultaneity of events is absolute.

The second, equally important feature of transformations for two reference frames is the absence of parameter (V). And if in the transformation for three reference systems (1), the velocity (V) predetermined the magnitude of the time scale offset ($\Delta t_A'$), which ensured the feasibility of transformations, then in the equations for two reference systems (6), the clock desynchronization parameters or the difference in their readings are initially set values (initial position).

The absence of parameter (V) in transformation (6) is explained by the absence of a privileged system (K_0) . This circumstance indicates that the classical Lorentz transformation (1), where this velocity is present, operates precisely with three reference frames (K_0), (K) and (K'), in which the system (K_0) acts as an absolute for the other two. This means that the speed of light for any reference frame moving relative to the center of the light wave emission (K_0) will always be relative. We should immediately note that the geometric center of the radiation of a light wave should not be confused with the source of its radiation, since the source after the radiation can move in space in any way.

The third feature of Lorentz transformations is their underlying mathematical technique of replacing coordinate (real) velocity with coordinate time, which by itself does not establish cause–and-effect relationships in physical phenomena and, as a result, cannot generate new physical laws. Therefore, coordinate transformations in one form or another that determine the relative location of two objects in space do not contain factors affecting the kinematics of each object, which indicates the absence of physical interaction between these objects.

Therefore, there is no reason to associate coordinate transformations with new physical laws, since transformations are just determining the mutual location of objects, and not the conditions for their interaction.

Note that any coordinate transformation is a comparison procedure, and therefore the comparison measures themselves – duration (time) and dimension (extent) – cannot change in any case during the measurement process. Moreover, these measures cannot be physical objects of study, since they are conventional measures, i.e. not an object, but a means for studying nature.

It should be noted that if the Lorentz transformations for two reference frames (6) are considered in the STR paradigm, then for ($k_S = k_S'$), «time dilation» can be defined as:

$$t' = \frac{t_0}{\sqrt{1 - \dfrac{v\,\Delta t'}{x + \Delta x'}}}$$

This example, using Lorentz transformations without the speed of light (c), once again shows that Einstein's interpretations of the properties of Lorentz transformations are fundamentally erroneous. Otherwise, it should be recognized that at a constant velocity (v) in a moving inertial reference frame (K'), the «time dilation» also depends on the readings of an incorrectly running clock ($\Delta t'$) and on the arbitrary displacement of the reference frames ($\Delta x'$) at the initial moment.

Perhaps relativistic scientists will have a different opinion on this example, which we have yet to hear.

Conclusions

The Lorentz transformations for two reference systems are Galilean transformations, which take into account additional factors such as the initial displacement of the reference systems, the direction of their movement, and the difference in clock readings, including the out–of-sync of their pace.

Coordinate transformations using the Lorentz method for two reference systems are always performed if the corrections for both forward (k_S) and reverse (k_S') transformations obey the Rule of simultaneity (5).

In the Lorentz transformation, measures of duration (seconds) and dimension (meters) are immutable measures regardless of the kinematic state of the reference systems under study.

In the Lorentz transformation, making a parameter invariant is a feature of the methodology used to compare coordinates and determine other parameters of these comparisons. The transfer of an invariant from measurement measures to their derivative does not generate new physical laws.

The exclusion of the speed of light (c) from transformations (6) did not affect the properties and feasibility of transformations when finding mutual coordinates.

Coordinate transformations using the Lorentz method are based on the concept of absolute simultaneity, otherwise coordinate transformations would not be possible.

This means that the simultaneity of events is absolute.

The problem of clock synchronization in judgments about the simultaneity of events in the STR is clearly far–fetched, since the Lorentz transformations take into account any nature of clock desynchronization located in different reference frames.

Transformations for two reference frames are applicable for both inertial and non-inertial reference frames.

The equations of coordinate transformations determine only the relative location of objects in space. There is no description of a physical phenomenon, i.e., the physical effect of objects on each other, in the Lorentz transformation. It follows that the Lorentz transformations cannot be the cause of any new physical laws.

Thus, coordinate transformations using the Lorentz method represent a mathematical apparatus in which, from the standpoint of classical physics, all kinematic parameters are consistent, consistent and logical. This suggests that the Lorentz transformations correspond to common sense and correctly reflect the world around us.

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