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Confinement, Meson Spectra and Pions $\pi^{\pm}$, π^{0} Mass Difference

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ABSTRACT

"Color confinement" of quarks is an unsolved problem in physics according to Wikipedia. This is explicitly resolved in the Scalar Strong Interaction hadron theory SSI here. Using SSI, the mass difference between charged $\pi^{\pm}$ and neutral π^{0} pions is estimated via a "marble" model to be =4.3 MeV, 6.5% off the measured 4.6 MeV, The measured charge radius $r_m \approx 0.66$ fm gives a far too small classical value $e^2/r_m \approx 2.2$ MeV. This large mass difference is mostly tied to the strong interaction potential between the quarks in their invisible relative space of quarks. The necessary background SSI material in the Appendix includes a meson mass formula that approximately accounts for ground state meson spectra, side-stepping "lattice" calculations on computers.

Keywords: scalar strong interaction hadron theory SSI, QCD, standard model, quark and meson equations of motion, relative space between quarks, hidden variables, confinement potentials in SSI, mass formula for meson spectra, "marble" model for $\pi^{\pm}$, pions $\pi^{\pm}$-π^{0} mass difference.

1. INTRODUCTION

The unsolved problem of the "proton radius puzzle" [1] has been explicitly resolved [2] in the Scalar Strong Interaction Hadron Theory SSI. The unsolved "dark energy problem" [1] has also been explicitly resolved in SSI [3 Ch 16] and [4].

Turning to the unsolved "color confinement problem" [1], its solution has been briefly sketched in [3 Sec. 4.3] and [4 Appendix]. This problem will be explained more specifically in Part I below in view of its half century long existence.

The so-considered equations of motion for mesons there are then applied to treat the unsolved problem of the large mass difference Δm_π=4.6 MeV [5] between the $\pi^\pm$ and π^0 mesons. This value far exceeds the classical charge mass of e^2/r_m =2.18 MeV where r_m =0.659 fm is the $\pi^\pm$ charge radius [5]. The SSI meson equations in Part I are extended to include a singlet gauge potential combined with a ”marble” model for $\pi^\pm$ to calculate Δm_π. This is treated in Part II.

The Appendix provides the development of the basic SSI meson equations including the mass formula for ground state meson spectra in agreement with data, rendering “lattice” calculations on computers unnecessary.

PART I RESOLVING THE ”COLOR CONFINEMENT PROBLEM”

2. Historical Example

Classical mechanics (CM), i. e., Newtonian mechanics with relativistic extension, has worked well before the atomic era. In atoms, however, the particle energies and angular momenta are much lower. In this region, CM fails. The last well-known application of CM to atomic phenomena is the Bohr-Sommerfeld model.

In such low particle energies and angular momenta regions, a new theory, quantum mechanics (QM), takes over. While one cannot go from CM to QM, which must be created, the reverse is possible. As the particle energy and angular momenta increase, QM merges into CM.

Here, quantum chromodynamics (QCD) works well at higher energies where it is perturbative; asymptotic freedom [6], [7] holds and quarks are color confined. As the quark energies decrease, asymptotic freedom ceases to hold, QCD becomes nonperturbative and confinement cannot be proved. During the past half century, many attempts have been made to extend such high energy QCD into the nonperturbative low energy region by means of lattice calculations on powerful computers. Although there are reported successes, these, unlike [3 Ch 5], have been unable to account for the basic meson spectra.

Usually, such simple problems are explained analytically. Computer calculations enter later in more complicated situations. Newton’s equations provide analytical solutions to the planet orbits around the sun, Computers are helpful when corrections due to interactions with other planets are included. In the atomic case, hydrogen atom has been accounted for analytically using the Schrödinger-Dirac equations. As the atoms get heavier and contain more electrons, computers are helpful to deal with the many-body problems. These took basically place in 3 decades after the inception of QM in the 1920´s.

Based upon analogy to the above case, QCD at higher energies cannot be pushed too far into the low energy, nonperturbative region. A new theory needs be created for this region, here the Scalar Strong Interaction hadron theory SSI [8], [3]. It has been successful in accounting for the ground state meson spectra in Table A1 below and many other hadronic phenomena like the above-mentioned proton radius puzzle and dark matter and energy [3 Ch 15-16]. Analogous to the merger of QM to CM, SSI gives at higher energies rise to a QCD Lagrangian [3 Ch 14].

3. QUARK AND MESON EQUATIONS AND CONFINEMENT IN SSI

The starting point is the van der Waerden equations [9], a transformed form of the Dirac equations that is manifestly Lorentz covariant and more suitable for relativistic particles. While Dirac´s wave functions are compatible with the regular representation of the Lorentz group, the van der Waerden spinors are basis vectors generating the fundamental representation of the SL(2,C) Lorentz group.

When applied to the motion of a quark A at x_I interacting with an antiquark B at x_{II} via the potential V_{SB}, these read [3 (2.1.1)-(2.2.5)]

$$\partial_I^{ab}\chi_{A\dot{b}}(x_I) - iV_{SB}(x_I)\psi_A^a(x_I) = im_A\psi_A^a(x_I) \tag{3.1a}$$

$$\partial_{Ibc}\psi_A^c(x_I) - iV_{SB}(x_I)\chi_{A\dot{b}}(x_I) = im_A\chi_{A\dot{b}}(x_I) \tag{3.1b}$$

$$\Box_I V_{SB}(x_I) = \tfrac{1}{2} g_s^2\left(\psi_B^b(x_I)\chi_{Bb}(x_I) + \psi_B^{\dot{b}}(x_I)\chi_{B\dot{b}}(x_I)\right) \tag{3.2}$$

Conversely, the antiquark B interacts with the quark A via V_{SA},

$$\partial_{II\dot{e}f}\chi_B^f(x_{II}) - iV_{SA}(x_{II})\psi_{B\dot{e}}(x_{II}) = im_B\psi_{B\dot{e}}(x_{II}) \tag{3.3a}$$

$$\partial_{II}^{d\dot{e}}\psi_{B\dot{e}}(x_{II}) - iV_{SA}(x_{II})\chi_B^d(x_{II}) = im_B\chi_B^d(x_{II}) \tag{3.3b}$$

$$\Box_{II} V_{SA}(x_{II}) = \tfrac{1}{2} g_s^2\left(\psi_A^b(x_{II})\chi_{Ab}(x_{II}) + \psi_A^{\dot{b}}(x_{II})\chi_{A\dot{b}}(x_{II})\right) \tag{3.4}$$

Here, ∂_I and ∂_{II} refer to differentiations with respect to x_I and x_{II}, respectively. g_s^2 is the scalar strong coupling constant for quark-antiquark interaction. m is the quark mass, $V_{SB}(x_I)$ is the scalar strong interaction potential at x_I generated by quark B at x_{II} and vice versa for $V_{SA}(x_{II})$. ψ and χ are the left-handed and right-handed, respectively, quark spinors and the dotted and undotted spinor indices *b, e, f*...run from 1 to 2. The subscripts A and B refer to the quark species.

The corresponding meson wave equations are obtained by multiplying (3.1), (3.2) with (3.3), (3.4). Mutual interaction between A and B causes the separable product wave functions to become nonseparable meson wave functions according to

$$\chi_{A\dot{b}}(x_I)\chi_B^f(x_{II}) \to \chi_{\dot{b}}^f(x_I, x_{II}), \psi_A^c(x_I)\psi_{B\dot{e}}(x_{II}) \to \psi_{\dot{e}}^c(x_I, x_{II}) \tag{3.5}$$

$$\chi_{A\dot{b}}(x_I)\psi_{B\dot{e}}(x_{II}) \to \chi_{\dot{b}\dot{e}}(x_I, x_{II}), \quad \psi_A^c(x_I)\chi_B^f(x_{II}) \to \psi^{cf}(x_I, x_{II}) \tag{3.6}$$

$$V_{SA}(x_{II})V_{SB}(x_I) \to \Phi_m(x_I, x_{II}) \tag{3.7}$$

Here, (3.5) represents the meson wave functions, (3.6) the antidiquark and diquark wave functions and (3.7) the scalar strong interaction potential between A and B. The above-mentioned multiplication gives rise to $3\times3=9$ equations containing the 2 meson wave functions in (3.5) and the interquark potential (3.7) as well a second group containing 2 quark (3.3), 2 diquark and 2 antidiquark (3.6) wave functions. Since this second group does not appear in meson equations, they are put to 0. Six of the 9 product equations drop out leaving behind the following 3 coupled meson equations [8], [3 (2.2.4-5)]

$$\partial_I^{a\dot{b}}\partial_{II\dot{e}f}\chi_{\dot{b}}^{f}(x_I,x_{II})+\big(m_A m_B-\Phi_m(x_I,x_{II})\big)\psi_{\dot{e}}^{a}(x_I,x_{II})=0 \tag{3.8a}$$

$$\partial_{I\dot{b}c}\partial_{II}^{d\dot{e}}\psi_{\dot{e}}^{c}(x_I,x_{II})+\big(m_A m_B-\Phi_m(x_I,x_{II})\big)\chi_{\dot{b}}^{d}(x_I,x_{II})=0 \tag{3.8b}$$

$$\Box_I\Box_{II}\Phi_m(x_I,x_{II})=-\frac{g_s^4}{4}\left(\psi^{b\dot{a}}(x_I,x_{II})\chi_{\dot{a}b}^{*}(x_I,x_{II})+\psi^{*a\dot{b}}(x_I,x_{II})\chi_{\dot{b}a}(x_I,x_{II})\right) \tag{3.9}$$

Carry out the transformation [3 (3.1.3a)]

$$x^{\mu}=x_{II}^{\mu}-x_I^{\mu},\qquad X^{\mu}=(1-a_m)x_I^{\mu}+a_m x_{II}^{\mu} \tag{3.10}$$

where a_m is a real constant. Conventionally, a_m=1/2 if the quark and antiquark have the same mass. Since x_I and x_{II} are invisible, these masses cannot be measured so that a_m is a free parameter at this stage. The meson laboratory coordinate X is observable but the relative coordinate x is a hidden variable. "Hidden" variable has been proposed by Einstein, Podolsky and Rosen in 1935 and D. Bohm in 1952 in connection with quantum mechanics, well before the quark era from the 1960´s and the dominating role it plays in SSI [3].

The interquark potential Φ_m depends only upon the distance $r=|\underline{x}|$ between both quarks. The meson wave functions on the right side of (3.9) contain the dependence $1/\Omega$, the inverse of the volume of the meson wave functions in the laboratory space $\underline{X}$. At rest, $\Omega\to\infty$ and the right side of (3.9) vanishes so that it reduces to [3 (3.2.8a)],

$$\Delta\Delta\Phi_m(r)=0,\qquad \Delta=\left(\partial/\partial\underline{x}\right)^2,\qquad \Phi_m(r)=d_m/r+d_{m0}+d_{m2}r^2 \tag{3.11}$$

where the d´s are integration constants. Choosing d_m=0 and $d_{m2}=-d_h^2$ [3 (3.2.21)], this term provides a confining potential that causes the wave functions ψ and χ in (3.8) to take the confined form (A7) below.

This confinement arises from the 4th order (3.9) and (3.11), which in their turn depend upon the multiplication of (3.1), (3.2) and (3.3), (3.4) in order to go from the invisible quarks to observable mesons via (3.10). This is a mathematical result in the hidden relative space $\underline{x}$ originating from the scalar quark-antiquark strong interaction potentials V_{SA} and V_{SB} at the quark level in (3.1-4). The last term in (3.11) in its turn leads to the simple mass formula (A8) which approximately agrees with data in Table A1.

If there were no such potentials, the meson equations (3.8) can be decoupled back to the quark equations (3.1) and (3.3). We would then not exist. That there is a material universe requires that quarks interact with each other, here via (3.2) and (3.4).

At higher energies, QCD color confinement may enter (end of Sec. 2). The inhomogeneous term on the right of (3.9) may also eventually enter and affect confinement.

4. CONFINEMENT OF BARYON WAVE FUNCTIONS [3 CH 10]

A ground state baryon consists of a quark interacting with a diquark, which replaces the antiquark in the treatment of mesons above. The 4^{th} order (3.11) is now replaced by the 6^{th} order [3 (10.2.2a)]. As these books may not be readily accessible, the main developments can also be seen in the open access [10],

$$\Delta\Delta\Delta\Phi_b(r) = 0, \qquad \Phi_b(r) = d_b/r + d_{b0} + d_{b1}r + d_{b2}r^2 + d_{b4}r^4 \tag{4.1}$$

where the d_b′s are again integration constants. When applied to data, however, it turns out that d_{b4}=0 so that the confining potential has the same form as that for mesons in (3.11), albeit with different value.

PART II PION $\pi^{\pm}$-π^0 MASS DIFFERENCE

The mass difference between the charged and neutral pions is Δm_π=4.5936 MeV [5] and has not been accounted for in any first principles′ theory. This value far exceeds the classical charge mass of e^2/r_m =2.18 MeV where r_m =0.659 fm is the $\pi^\pm$ charge radius [5]. The predicted $\pi^\pm$–π^0 mass difference 2.54 MeV in Table A1 below, like 2.1 MeV [3 (5.1.2)] used to represent the charge contribution to meson mass, are both also far less than data 4.6 MeV.

This indicates that the approximate (A8), mentioned near the end of Sec. 3, is incomplete with respect to electromagnetic corrections to the meson masses. Here, Δm_π will be estimated from (3.3-4). In Sec, 5, a "Marble" model for $\pi^\pm$ is proposed. The SSI meson equations (3.8) are generalized to include electromagnetic gauge fields as perturbations. The procedure is analogous to similar generalizations in [3 Ch 6-7]. As these books may not be easily accessible, the main developments can also be seen in the open access [11]. These are then applied to pions in Sec. 6, where Δm_π is estimated. The results are discussed in Sec. 7 and 8. Some necessary underlying material in SSI are provided in the Appendix.

5. THE "MARBLE" MODEL FOR $\pi^\pm$

The pion beta decay $\pi^- \rightarrow e + \bar{\nu}_e$ suggests that $\pi^\pm$ and π^0 have the same strong interaction, the mutual interaction of the u and d quarks, irrespective their charges. This strong attraction takes place in the relative, "hidden" space x^μ between the quarks separated from the electromagnetic interactions between quarks in different hadrons in the visible laboratory space X^μ (3.10).

SSI contains both x^μ and X^μ intermixed and the general problems are rather complicated. So far, only problems containing x^μ and X^0, the laboratory time, in the simple form of $exp(-iE_0X^0)$, have been treated. Here, E_0 is the π^0 mass and the laboratory space $\underline{X}$ does not enter.

There is only one data point in this problem, namely r_m=0.659 fm [5] in $\underline{X}$ space. The simplest assumption is to represent $\pi^\pm$ as a "marble" with this radius evenly filled with a charge $\pm e$ in $\underline{X}$ space. Now the measured r_m=0.659 fm is generally some mean value of many measurements corresponding to different marble sizes. Assume that these charge distributions are normally distributed, they are converted to a "marble" having sharp boundaries with radius $R_m= r_m\sqrt{\pi/2}$=0.584 fm [12 (2.1)]. This leads to a mass difference of 2.47 MeV which is still much less than Δm_π.

In the marble model, the form in $\underline{X}$ mentioned above (A3) is replaced by a step function

$$\Psi(|\underline{X}| = R) = 1(\text{unit length})^{(-3/2)} \quad \text{for } R<R_m \qquad \text{and } 0 \qquad \text{for } R \geq R_m \tag{5.1}$$

while keeping the time dependence $exp(-iE_0X^0)$. The ansatz (A3) is then replaced by

$$\psi^{a\dot{e}}(\underline{x}) \to \psi^{a\dot{e}}(\underline{x})\, \Psi(R) exp(-iE_0X^0) \to \delta^{a\dot{e}}\psi_0(r)\, \Psi(R) exp(-iE_0X^0)$$

$$\chi_{bf}(x) \to \chi_{bf}(\underline{x})\, \Psi(R) exp(-iE_0X^0) \to \delta_{bf}\chi_o(r)\, \Psi(R) exp(-iE_0X^0) \tag{5.2}$$

where $r=|\underline{x}|$. The effect of quark charges, at first for π^+, can now be introduced via the conventional minimal substitutions

$$\partial_I^{ab} \to \partial_I^{ab} + \frac{1}{2}q_u A^{ab}(X), \qquad \partial_{II}^{f\dot{e}} \to \partial_{II}^{f\dot{e}} - \frac{1}{2}q_d A^{f\dot{e}}(X)$$

$$\partial_{I\dot{g}a} \to \partial_{I\dot{g}a} + \frac{1}{2}q_u A_{\dot{g}a}(X), \qquad \partial_{II\dot{e}d} \to \partial_{II\dot{e}d}^{f\dot{e}} - \frac{1}{2}q_d A_{\dot{e}d}(X) \tag{5.3}$$

in (A1). q_u =2e/3 and q_u =$-e$/3. As there is no magnetic field only the time component $A_0(\underline{X})$ of the vector potential enters. In the marble model, $A_0(\underline{X}) \to A_0(R)$ satisfies

$$\left(\frac{\partial}{\partial \underline{X}}\right)^2 A_0(R) = 4\pi\rho\Psi^2(R) = \frac{3(q_u - q_d)}{R_m^3}, \quad A_0(R) = \frac{e}{2R_m^3}R^2 \text{ for } R < R_m \text{ and } \frac{e}{2R_m} \text{ for } R \geq R_m \tag{5.4}$$

where ρ is the charge density in the marble and e is the meson charge. With the introduction of $A_0(R)$, the meson energy E_0 in (5.2) becomes E_0+E_1, where E_1 is the effect of the perturbation A_0. With these preliminaries and (3.10), the differential operators in (A1) can be written as

$$\partial_I^{ab} \to D_I^{ab} = \left[\frac{i}{2}\delta^{ab}\left(E_0 + E_1 + q_u A_0(R)\right) + \underline{\sigma}^{ab}\left(-\frac{\partial}{2\partial \underline{X}} + \frac{\partial}{\partial \underline{x}}\right)\right] \tag{5.5a}$$

$$\partial_{II}^{f\dot{e}} \to D_{II}^{f\dot{e}} = \left[\frac{i}{2}\delta^{f\dot{e}}\left(E_0 + E_1 - q_d A_0(R)\right) + \underline{\sigma}^{f\dot{e}}\left(-\frac{\partial}{2\partial\underline{X}} - \frac{\partial}{\partial\underline{x}}\right)\right] \tag{5.5b}$$

$$\delta_{II\dot{e}d} \to D_{II\dot{e}d} = \left[\frac{i}{2}\delta_{\dot{e}d}\left(E_0 + E_1 - q_d A_0(R)\right) + \underline{\sigma}_{\dot{e}d}\left(\frac{\partial}{2\partial\underline{X}} - \frac{\partial}{\partial\underline{x}}\right)\right] \tag{5.5c}$$

$$\delta_{I\dot{g}a} \to D_{I\dot{g}a} = \left[\frac{i}{2}\delta_{\dot{g}a}(E_0 + E_1 + q_u A_0(R) + \underline{\sigma}_{\dot{g}a}\left(\frac{\partial}{2\partial\underline{X}} + \frac{\partial}{\partial\underline{x}}\right)\right] \tag{5.5d}$$

Eqs. (A1) with (5.2) now becomes

$$D_I^{ab}\delta_{bf}\chi_o(r)D_{II}^{f\dot{e}} - \left(M_m^2 - \Phi_m(r)\right)\delta^{a\dot{e}}\psi_0(r) = 0 \tag{5.6a}$$

$$D_{I\dot{g}a}\delta^{a\dot{e}}\psi_0(r)D_{II\dot{e}d} - \left(M_m^2 - \Phi_m(r)\right)\delta_{\dot{e}d}\chi_o(r) = 0 \tag{5.6b}$$

The common factor $\Psi(R)exp\left(-i(E_0 + E_1)X^0\right)$ attached to the ψ_0 and χ_o, as in (5.2), has been dropped. The *D* operators are arranged such that the spinor indices couple to each other sequentially with the convention that operators on the right side of the wave functions $\chi_o(r)$ and $\psi_0(r)$ operate backwards towards left on them.

Since the D_I and D_{II} operators now depend also upon the visible laboratory $\underline{X}$, they no longer commute as the ∂_I and ∂_{II} do in (A1). Thus, the order of operation makes difference. For example, performing the D_I operation first followed by D_{II} in (5.6) will lead to different result from that obtained by $D_I \leftrightarrow D_{II}$.

Following the example of calculation of magnetic moment from the Dirac equation, operate (5.6a) by $D_{I\dot{g}a}$ from the left and $D_{II\dot{e}d}$ from the right leads to

$$D_{I\dot{g}a}D_I^{ab}\delta_{bf}\chi_o(r)D_{II}^{f\dot{e}}D_{II\dot{e}d} - D_{I\dot{g}a}\left(M_m^2 - \Phi_m(r)\right)\delta^{a\dot{e}}\psi_0(r)D_{II\dot{e}d} = 0 \tag{5.7}$$

There are now 4 possible combinations:

$$D_{I\dot{g}a}\left\{\left[\left(D_I^{ab}\delta_{bf}\chi_0(r)\right)D_{II}^{f\dot{e}}\right]D_{II\dot{e}d}\right\} + D_{I\dot{g}a}\left\{(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)D_{II\dot{e}d}\right\} = 0 \tag{5.8a}$$

$$D_{I\dot{g}a}\left\{\left[D_I^{ab}\left(\delta_{bf}\chi_0(r)D_{II}^{f\dot{e}}\right)\right]D_{II\dot{e}d}\right\} + D_{I\dot{g}a}\left\{(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)D_{II\dot{e}d}\right\} = 0 \tag{5.8b}$$

$$\left\{D_{I\dot{g}a}\left[\left(D_I^{ab}\delta_{bf}\chi_0(r)\right)D_{II}^{f\dot{e}}\right]\right\}D_{II\dot{e}d} + \left\{D_{I\dot{g}a}(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)\right\}D_{II\dot{e}d} = 0 \tag{5.9a}$$

$$\left\{D_{I\dot{g}a}\left[D_I^{ab}\left(\delta_{bf}\chi_0(r)D_{II}^{f\dot{e}}\right)\right]\right\}D_{II\dot{e}d} + \left\{D_{I\dot{g}a}(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)\right\}D_{II\dot{e}d} = 0 \tag{5.9b}$$

The operations in the parentheses (…) are performed first followed by those in brackets […] and then those in the braces {...}.

To zeroth order, (5.8) is equivalent to (A1) which led to the ground state meson spectra predictions in Table A1 [3 Ch 5].

6. FIRST ORDER TERMS AND PREDICTED RESULTS

Our task is to obtain the dependence of the first order E_1 on qA_0 from (5.8-9). This dependence takes place in the laboratory space $\underline{X}$ so that the hidden $\underline{x}$ dependence can be removed by carrying out the contraction

$$\int d\underline{x}^3\ \delta^{d\dot{g}}\ \chi_0(r)(5.8, 5.9)/\int d\underline{x}^3\ \chi_0^2(r) \qquad (6.1)$$

Using the Dirac formula

$$\left(\underline{\sigma}^{a\dot{b}}\underline{a}\right)\left(\underline{\sigma}_{\dot{b}c}\underline{b}\right) = \delta_c^a\left(\underline{a}\underline{b}\right) + i\underline{\sigma}_c^a\left(\underline{a}\times\underline{b}\right) \qquad (6.2)$$

the bracketed expressions in (5.8a) and (5.8b) become respectively

$$\left[\left(D_I^{ab}\delta_{bf}\chi_0(r)\right)D_{II}^{f\dot{e}}\right] =$$

$$\delta^{a\dot{e}}\left[-\left(M_m^2-\Phi_{mav}\right)-E_0\left(\frac{E_1}{2}+\frac{e}{8R_m^3}(q_u-q_d)R^2\right)\right]+\underline{\sigma}^{a\dot{e}}\frac{i}{2}\left[-q_u\underline{X}-\frac{e}{2R_m^3}(q_u+q_d)R^2\frac{\partial}{\partial\underline{x}}\right] \qquad (6.3a)$$

$$\left[D_I^{ab}\left(\delta_{bf}\chi_0(r)D_{II}^{f\dot{e}}\right)\right] =$$

$$\delta^{a\dot{e}}\left[-\left(M_m^2-\Phi_{mav}\right)-E_0\left(\frac{E_1}{2}+\frac{e}{8R_m^3}(q_u-q_d)R^2\right)\right]+\underline{\sigma}^{a\dot{e}}\frac{i}{2}\left[q_d\underline{X}-\frac{e}{2R_m^3}(q_u+q_d)R^2\frac{\partial}{\partial\underline{x}}\right]$$

(6.3b)

$$\Phi_{mav} = \int d\underline{x}^3\ \chi_0^2(r)\Phi_m(r)/\int d\underline{x}^3\ \chi_0^2(r) = 0.5361\ \ GeV^2 \qquad (6.4)$$

Eqs. (6.3a) and (6.3b) differ only in the triplet term in which $q_u \rightarrow -q_d$, just like that between (5.5a) and (5.5b).

In (6.4), terms linear in $\partial/\partial\underline{x}$ drop out due to (6.1). Terms containing Δ are removed via (A5). The first order terms in (5.8) are collected to singlet terms $\delta_{\dot{g}d}\ (\ldots);$ the triplet terms $\underline{\sigma}_{\dot{g}d}\ (\ldots)$ drop out also due to (6.1). Collecting the first order E_1 and q terms, it was found, after some algebra, the mass differences

$$\Delta m_\pi\ =\ E_1 = -\frac{e}{R_m^3}\left(\frac{q_u+q_d}{8\left(M_m^2-\Phi_{mav}\right)}+\frac{q_u-q_d}{4}R^2\right) = 4.306\ \text{MeV} \quad \text{for } R^2 \rightarrow (R^2)_{\text{av}} = R_m^2/2 \qquad (6,5a)$$

$$\Delta m_\pi\ =\ E_1 = -\frac{e}{R_m^3}\left(-\frac{q_u+q_d}{8\left(M_m^2-\Phi_{mav}\right)}+\frac{q_u-q_d}{4}R^2\right) = -4.922\ \text{MeV} \quad \text{for } R^2 \rightarrow (R^2)_{\text{av}} = R_m^2/2 \qquad (6,5b)$$

for the sequences (5.8a) and (5.8b), respectively. Similarly, (5.9a) and (5.9b) leads to the same results but with (6.5a)↔(6.5b).

The predicted -4.922 MeV *is* rejected. The predicted 4.3 MeV is close to the measured value of 4,6 MeV, differing by 7%. The result (6.5a) was derived for π^+ but also holds for π^- as it depends only upon the square of the charges. If the R^2 in (6.5) $\to -4R_m^2$, it contributes the usual classical e^2/r_m =2.18 MeV mentioned in the Introduction. This term originates from the Laboratory $\underline{X}$ and is small compared to the term $\propto (q_u + q_d)$, which originates in the hidden, relative space $\underline{x}$.

7. EVALUATION OF THE RESULTS AND IMPLICATIONS

The presence of $|\underline{X}|=R$ in (6.5) comes from the potential $A_0(R)$ in (5.4) and reminds one of that the $\underline{X}$ dependence of (5.6) remains not dealt with. $\partial/\partial\underline{X}$ operating on the step function (5.1) vanishes for $R< R_m$ and $R>R_m$ but diverges for $R=R_m$ on the surface of the marble. This problem has not been investigated due to its difficulty. These show that this marble model is not fully compatible with the basic equations (5.6). Thus, this model can only provide an estimate of the mass difference Δm_π, not an exact prediction. However, this estimate should be rather good because the R^2 term, approximated by its mean value of $R_m^2/2$ in (6.5), makes up only ~6% of Δm_π.

Δm_π is largely controlled by the strong interaction Φ_{mav} in the hidden space. This large Φ_{mav} is also responsible for the low pion masses, as is shown beneath Table A2. Further, Δm_π depends 94% upon $q_u+q_d=e/3$ only 4% upon the conventional $q_u-q_d=e$ in (6.5). This shows that the strong interaction between the quarks also controls Δm_π. Our present electrostatic conception of Δm_π is no longer valid.

As it can be seen above Sec. 7, this 4% parts is 8 times smaller and of opposite sign relative to the classical value e^2/R_m. It may also be noted that if the first − sign in (6.5) is changed to +, then (6.5a) will give −4.306 and (6.5b) +4.922 Mev, respectively. The mean value of 4.922 and 4.306 is 4.6145 Mev, only 0.45% off the measured 4.5936 MeV. This is equivalent to dropping the R^2 terms originating in the $\underline{X}$ space in (6.5). Only the hidden space part physics contributes; the laboratory space contribution is averaged out.

If the anti-*d* quark in (6.5) is replaced by an anti-*s* quark, it may formally be applied to the kaon system $K^\pm$-K^0. (6.5) yields a correction of $\approx\pm10$ MeV. It is bigger than the predicted 6.1 MeV and measured 3.93 MeV in Table A1 below. But these values are of the same magnitude and bigger than those for the pions. A basic difference here is that, while $\pi^\pm$ and π^0 have the same quark content and same strong interaction, as was mentioned in Sec. 5, $K^\pm$ and K^0 have different quark contents, contains a heavier *s* quark and hence perhaps have somewhat different strong interactions. Further, these kaon masses have already been used there to fix the quark masses in Table A2 and to determine the predicted values in Table A1 and hence cannot be changed. While π^0 due to its small mass has not been used for those purposes. Thus, (6.5) cannot be applied to the kaons without additional investigation. These remarks also apply to the D and B mesons in Table A1.

Eq. (6.5a) gives a far greater value than the phenomenological 2.1 MeV assigned to the charge contribution to the 0^- meson and quark masses in Tables A1 and A2 below. There is presently an uncertainty in the charge corrections to the strong interaction (A8) in these tables.

8. CONCLUSION

Half century after the advent of “asymptotic freedom” [5, 6], “confinement” has still not been proven. “Color confinement” is listed by Wikipedia [1] as an unsolved problem in physics. This has been implicitly resolved in [3 Ch 3-5] and briefly sketched in [3 Sec. 4.3] in SSI. Here, this has been treated explicitly in Part I.

The equations of motion for mesons given there also led to a mass formula (A8) that approximately accounts for the ground state meson spectra in Table A1, sidestepping complicated lattice calculations on computers. The treatment takes place in the invisible relative space between the quark and the antiquark in which they interact via strong, confining potentials given there. The electrostatic contributions have been included phenomenologically. These have been given in Part I and the Appendix.

To treat the electrostatic contributions more formally, the same meson equations are generalized to include a singlet gauge field caused by the quark charges in Part II. A “marble” model for the charged $\pi^{\pm}$ was adopted. This model leads to a $\pi^{\pm}$- $\pi^{\pm}$ mass difference of Δm_{π}=4.3 MeV, close to the measured 4.6 MeV [5].

The above three topics, Part I, Part II and Appendix are seen to be interconnected and their results support each other.

Outside the meson sector, SSI can also account for nuclear force [12], dark matter, dark energy and antigravity in expanding universe [4].

9. ADDENDUM TO REF. 12

In this recent reference, the origin of nuclear force is assigned to the Coulomb attraction between $u(d)$ quark in a proton and $d(u)$ quark in a neighboring neutron, without providing explicit experimental support. This is remedied here. Firstly, eq. (3.1) there shows that the deuteron binding energy ≈2.2 MeV is correctly estimated; no “strong” force is involved. Since nuclei consist mostly of such pairs, such an assignment provides the basic mechanism of nuclear bond. As more nucleons are added in heavier nuclei, the two body force in the deuteron has to be extended to include genuine many body interactions. Further, the "quark sharing" mechanism in §4.1 enters. These have been estimated to lead to a binding energy of up to 14 MeV per nucleon, of the same magnitude as the observed maximum of 8.9 MeV.

The deuteron can be disintegrated by a ≈2.2 MeV photon. Photonfission of the actinides takes place in the low 10´s MeV. These support the present Coulomb assignment. In SSI, strong *u-d* binding takes place inside a nucleon in "hidden relative space", like *x* in (3.10) with the antiquark there replaced by a diquark. This binding prevents the disintegration of nucleon but is of the order of GeV, far too big for nuclear binding

APPENDIX SSI MESON EQUATIONS AND RESULTS

The equations of motion of mesons (3.8) becomes [3 (2.3.22), (2.4.1)]

$$\partial_I^{ab}\partial_{II}^{f\dot{e}}\chi_{bf}(x_I,x_{II})-\left(M_m^2-\Phi_m(x_I,x_{II})\right)\psi^{a\dot{e}}(x_I,x_{II})=0 \qquad \text{(A1a)}$$

$$\partial_{Ibc}\partial_{II\dot{e}d}\psi^{c\dot{e}}(x_I,x_{II})-\left(M_m^2-\Phi_m(x_I,x_{II})\right)\chi_{bd}(x_I,x_{II})=0 \qquad \text{(A1b)}$$

$$m_Am_B\rightarrow M_m^2=\frac{1}{4}(m_1+m_2)^2 \qquad \text{(A2)}$$

For free mesons, (3.10) will cause χ and ψ to contain X^μ dependence in the form of $*(1/\Omega)exp(i\underline{K}\underline{X}-iE_0X^0)$ where E_0 is the meson mass, $\underline{K}$ its momentum which $\rightarrow 0$ in the rest frame. The normalization volume $\Omega\rightarrow\infty$ in [3 (3.1.5-7, 9)]. This ansatz removes the $\underline{X}$ dependence. Since the pions are pseudoscalar, they are represented by the singlets χ_0 and ψ_0 (A5) below. The vector part of the wave function $\underline{\psi}$ and $\underline{\chi}$ representing the vector mesons are dropped. The x^μ dependence part read [3 (3.1.9), 4 (A6)]

$$\psi^{c\dot{e}}(\underline{x})=\delta^{c\dot{e}}\psi_0(\underline{x})\,exp\,(i\omega_0x^0)$$

$$\chi_{bf}(x)=\delta_{bf}\chi_o(\underline{x})\,exp\,(i\omega_0x^0), \qquad \text{(A3)}$$

where ω_0 is the relative energy between the quarks.

The two unknown parameters a_m and ω_0 must cancel each other via the relation

$$a_m=1/2+\omega_0/E_0 \qquad \text{(A4)}$$

given in [8] and [3 (3.1.10a), 4 (A6)]. During the short life of the pions, ω_0=0 and a_m=1/2. For protons in hydrogen atoms in rarified galactic space, a_m<1/2 or >1/2 causes the relative energy ω_0 between the diquark and quark of proton to become dark matter or dark energy, respectively [4], [3 Ch 15-16].

The interquark strong potential Φ_m in (A1) satisfies (3.11), [3 (3.1.14)]. Hence, χ_0 and ψ_0 are also functions of r. With these preliminaries, $\partial_I = \partial/\partial x_I$ and $\partial_{II} = \partial/\partial x_{II}$ in (A1) can via (3.10) be expressed in terms of X^μ and x^μ and (A1) can be reduced to the pseudoscalar meson radial equation [3 (3.2.3b, 3.2.8a, 3.2.21), Table 5.2]

$$\left(\Delta + E_0^2/4 + \Phi_m(r) - M_m^2\right)\chi_0(r) = 0, \qquad \psi_0(r) =- \chi_0(r) \tag{A5}$$

$$\Phi_m(r) = d_{m1}/r + d_{m0} - d_h^2 r^2, \qquad d_{m0} = 0.64113\text{GeV}^2, \qquad d_h = 0.07\text{GeV}^2 \tag{A6}$$

$$\chi_0(r) = \frac{1}{\sqrt{\Omega}}\alpha_{00}\, exp\left(-\frac{d_h}{2}r^2\right), \alpha_{00} = \left(\frac{d_h}{\pi}\right)^{3/4} = 0.0577\text{GeV}^{3/2} \tag{A7}$$

The size <r> of pseudoscalar mesons in the hidden space found using $(\chi_0)^2$ is ≈5.3 fm, about 73% greater than the nucleon size of 3.05 fm [3 (12.6 22)], [12 (A17)].

The three d constants in (A6) are originally unknown integration constants in the solution of the 4th order (3.11). However, (A5) only allows one r dependent term in (A6) for discrete, terminated series solutions. The d_{m1} term was chosen first but was later replaced by the d_h term which leads to confinement (A7). (A1) then via (A5) yields the mass formula [3 (5.1.1)]

$$E_J = \sqrt{\left(m_p + m_r\right)^2 - 4d_{m0} + 8d_h\left(J + \frac{3}{2}\right)} \tag{A8}$$

where J=0 and 1 refers to pseudoscalar and vector mesons, respectively. The subscripts denote quark species. (A8) is a purely strong interaction result from the hidden space and is independent of the quark charges. d_{m0} and 5 quark masses in Table A2 are to be fixed by 6 pseudoscalar meson masses in Table A1. (A1) via (A5) then leads to Table A1 and are to be compared with the E_{m0}–2.1 and E_{m1} lines there.

Table A1. E_{m0} and E_{m1} are data [5] in MeV. 2.1 MeV is correction due to meson charge [3 (5.1.2)] assuming the same charge radius for all charged 0⁻ mesons. $2d_h = (E_{m1}^2-(E_{m0}-2.1)^2)/4$ and has been taken to be 0.14 GeV^2 as an average. The * marked pseudoscalar mesons are used as input to fix the quark masses shown in Table A2 [3 Table 5.2].

	*$\pi^\pm$	π^0	*$K^\pm$	*K^0	$D^\pm$	*D^0	*D_s^+	$B^\pm$	*B^0	B_s^0
E_{m0}	139.57	134.98	493.68	497.61	1869.7	1864.8	1968.4	5279.3	5279.7	5366.9
E_{m0}–2.1	137.47	134.98	491.58	497.61	1867.6	1864.8	1966.3	5277.2	5279.7	5366.9
(A8)	139.04	137.52	491.86	497.96	1867.3	1864.7	1966.4	5276.9	5279.1	5363.3
	ρ^+	ρ^0	$K^{*\pm}$	K^{*0}	D^{*+}	D^{*0}	D_s^{*+}	B^*	B^*	B_s^{*0}
E_{m1}	775.1	775.1	891.76	895.6	2010.3	2006.9	2112.2	5324.7	5325	5415

(A8)	761.1	761.1	895.5	898.8	2011.7	2009.2	2104	5329.6	5331.8	5415.2
$2d_h$	0.1453	0.1453	0.1379	0.1386	0.1364	0.1375	0.1467	0.1202	0.1i94	0.1308

Eq. (A8) values are sensitive to the choice of $2d_h$. If the chosen 0.14 is smaller by 0.05%, the (A8) value 139.04 for $\pi^{\pm}$ will decrease to 137.47 in agreement with data. The large difference between (A8) and data E_{m1} for ρ may be due to its large width. One also sees that the flavor independence of $2d_h$ holds well despite the large mass differences between the π and the B_s mesons. The predicted $\pi^{\pm}$-π^0 mass difference ≈ 1.52 MeV is much smaller than data 4.6 MeV and is the subject of Part II.

Table A2. Quark masses and d_{m0} obtained from data in Table A1

m_u (GeV)	m_d-m_u	m_s	m_c	m_b	d_{m0} (GeV^2)
0.6592	0.00215	0.7431	1.6215	4.7786	0.64113

The relatively large d_{m0} value nearly balances off the other two positive terms in (A8) for the π´s and makes their mass much smaller their vector counterpart, the ρ´s. As the quark masses increase, towards the right in Table A1, this term becomes less important such that the B^* and B masses differ insignificantly.

Deviations of the (A8) predictions and data in Table A1 may partly be due to similar mechanism for Δm_π in (6.5a), as was discussed at the end of Sec. 7. The validity of the phenomenological assumption of 2.1 MeV for the charge contribution to 0^- mesons has not been proven. Also, the kaon masses have been chosen to be inputs in (A8) and are fixed. So, charge corrections to the strong interaction results (A8) in Tables A1 and A2 are uncertain.

Nevertheless, the relatively good agreement of (A8) with data in Table A1 indicates that lattice calculations on computers mentioned in Sec. 2 can be avoided.

ERRATUM: “Weyl” should read “van der Waerden” in [12], [2]

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